Nassau County Interscholastic Mathematics League

Contest #1 Answers must be integers from 0 to 999, inclusive. 2016 – 2017

No calculators are allowed.

Time: 10 minutes

1) A clock chimes 4 times at 4 o'clock. It takes 6 seconds from the first chime to the last chime. At the same constant rate, compute the number of seconds from the first chime to the last chime at 12 o'clock.

2) Compute the sum of the roots of $3^{2x} - 4 \cdot 3^{x+1} = -27$.

Time: 10 minutes

3) A linear function is defined by $f(x+5) = f(x) + 100$. Compute the slope of the linear function $f(x)$.

4) The measure of one angle of ΔABC, which is inscribed in circle O, is $47°$. The measures of the three angles of ΔABC form an arithmetic sequence. If the length of the second longest side of ΔABC is 12 and the radius of circle O, in simplest radical form, is $p\sqrt{q}$, compute pq.

Time: 10 minutes

5) If $a^2 + b^2 = 15$ and $ab = 5$, compute the positive value of $a + b$.

6) In isosceles ΔRST, point M is the midpoint of altitude $\overline{RL}$ to base $\overline{ST}$. Lines $\overleftrightarrow{TM}$ and $\overleftrightarrow{SM}$ intersect $\overline{RS}$ and $\overline{RT}$ in points H and K, respectively. The ratio of the area of quadrilateral $RHMK$ to the area of ΔRST may be expressed in simplest form as $\frac{p}{q}$. Compute $p + q$.

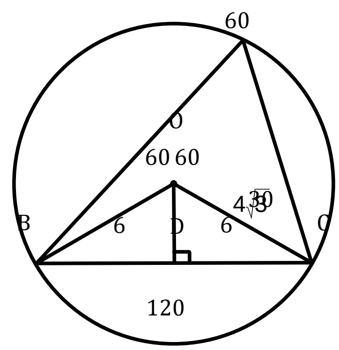
Solutions for Contest #1

1) When the clock strikes 4 o'clock there are 3 two-second intervals between strikes. So, for the clock to strike 12 o'clock, there are 11 two-second intervals. Therefore, it takes **22** seconds for this clock to strike 12 o'clock.

2) Re-write the given equation as $3^{2x} - 4 \cdot 3^{x+1} + 27 = 0 \rightarrow 3^{2x} - 12 \cdot 3^x + 27 = 0$. Now let $a = 3^x$. So we have $a^2 - 12a + 27 = 0 \rightarrow (a-9)(a-$ *3=0→3x−93x−3=0→3x=9* or *3x=3→x=2* or *x=1*. The required sum is **3.**

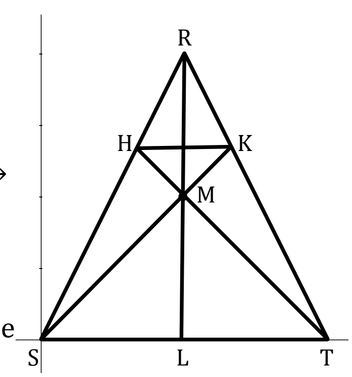
3) Since f is linear, $f(x) = mx + b$. So, $f(x+5) = m(x+5) + b = mx + b + 100 \longrightarrow mx + 5m + b = mx + b + 100 \longrightarrow 5m = 100 \longrightarrow m = \mathbf{20}$.

4) Let the measures of the angles of ΔABC in degrees be $47, 47+d,$ and $47+2d$. Setting their sum equal to 180 yields $d = 13$. The degree-measures of the angles of ΔABC are 47, 60, and 73. If $m\angle A = 60°$, draw radii $\overline{OB}$ and $\overline{OC}$ to form isosceles ΔBOC with $m\angle BOC = 120°$. Let $\overline{OD}$ bisect $\angle BOC$ forming two 30°-60°-90° congruent triangles. Since $BC = 12$ and $DC = 6$, radius $OC = 4\sqrt{3}$ and the required product is **12.**



5) Start with $(a+b)^2 = a^2 + b^2 + 2ab = 15 + 10 = 25$. So $a + b = \pm 5$. The required value is **5.**

6) Let $RM = ML = b$ and let $SL = LT = a$. Place ΔRST in the coordinate plane with S (0,0), $L\ (a,0), T(2a,0), M(a,b)$ and $R\ (a,2b)$, where $a > 0$ and $b > 0$. Then, the equation of $\overleftrightarrow{SMK}$ is $y = \frac{b}{a}x$ and the equation of $\overleftrightarrow{RKT}$ is $y - 0 = \frac{-2b}{a}(x - 2a)$. Solve the system of equations to get the coordinates of point K: $\frac{b}{a}x = \frac{-2b}{a}(x-2a) \rightarrow x = -2x + 4a \rightarrow 3x = 4a \rightarrow x = \frac{4a}{3}$ and $y = \frac{4b}{3}$. By symmetry, the coordinates of H are $\left(\frac{2a}{3}, \frac{4b}{3}\right)$. Since $\Delta RHM \cong \Delta RKM$ by ASA and $\overline{RH} \cong \overline{RK}$ and $\overline{HM} \cong \overline{KM}$, $RHMK$ is a kite. The area of the kite is $\frac{1}{2}(HK)(RM) = \frac{1}{2}\left(\frac{2a}{3}\right)(b) = \frac{ab}{3}$. The area of $\Delta RST = \frac{1}{2}(2a)(2b) = 2ab$. So, the required ratio is $\frac{ab}{3}/2ab = \frac{1}{6}$ and the required sum is **7.**



Alternatively, use mass points. The masses at S and T are each 1. So, L and R are each 2, M is 4, and H and K are each 3. So, let $RH = RK = d, HS = KT = 2d$, and $RM = ML = b$. Let $m\measuredangle SRL = \theta$. Therefore, $\frac{(RHMK)}{(RST)} = \frac{(RHM)}{(RSL)} = \frac{\frac{1}{2}bd\sin\theta}{\frac{1}{2}\cdot 2b\cdot 3d\sin\theta} = \frac{1}{6}$. The required sum is **7.**