Nassau County Interscholastic Mathematics League

Contest #4 Answers must be integers from 0 to 999, inclusive. 2014 – 2015

Calculators are allowed.

Time: 10 minutes

1) What two-digit number is exactly twice the product of its digits?

2) Jake brought a $100 voucher for tickets to a box office and asked for $5 tickets, $2 tickets, and $1 tickets. He received at least one of each and got 10 times as many $1 tickets as $2 tickets. How many $5 tickets did he receive?

Time: 10 minutes

3) Two natural numbers differ by 3 and their squares differ by 273. Compute the smaller of the two numbers.

4) Line segments $\overline{PQ}$, $\overline{PR}$, and $\overline{PS}$ are three edges of a cube. If $PQ = 2$ and the area of ΔQRS is expressed in simplest form as $p\sqrt{q}$, compute $p + q$.

Time: 10 minutes

5) If the length of an edge of a cube is increased by 50%, the cube's surface area is increased by n%. Compute n.

6) In $\Delta PQR, PQ = 4$, and $QR = 10$. Point M is the midpoint of $\overline{PQ}$, point N is the midpoint of $\overline{PR}$, and angle bisector $\overrightarrow{QS}$ meets $\overline{MN}$ in point T. If the ratio MT/TN is expressed in simplest form as x/y, compute $x + y$.

Solutions for Contest #4

1) If t is the tens digit and u is the ones digit, then $10t + u = 2tu \longrightarrow 10t = 2tu - u \longrightarrow$ $10t = u(2t-1) \longrightarrow u = \frac{10t}{2t-1}$. The only solution in which both t and u are digits is $t = 3$ and $u = 6$. Therefore the required number is **36.**

2) Suppose he received a \$1 tickets, b \$2 tickets, and c \$5 tickets. Then, $a + 2b + 5c = 100$ and $a = 10b$. So, $12b + 5c = 100$. Notice that b must be a multiple of 5. But, if $b = 10$, then $c < 0$. So, $b = 5$, $a = 50$, and $c = 8$. The answer is **8**.

3) If $x - y = 3$ and $x^2 - y^2 = 273$, then $(x-y)(x+y) = 3(x+y) = 273$ and $x + y = 91$. Adding the latter equation to the original linear equation yields $2x = 94 \longrightarrow x = 47$ and
$y = 44$. The required number is **44.**

4) Since each of $\overline{PQ}, \overline{PR}$, and $\overline{PS}$ is a diagonal of a face of the cube, ΔQRS is equilateral. In isosceles right ΔPQR, $QR = 2\sqrt{2}$, so the area of ΔQRS is $\frac{s^2\sqrt{3}}{4} = \frac{(2\sqrt{2})^2\sqrt{3}}{4} = 2\sqrt{3}$ and the required sum is **5**.

5) If the original surface area is $6x^2$, then the new surface area is $6(1.5x)^2 = 13.5x^2$. The percent increase is $\frac{13.5x^2 - 6x^2}{6x^2} = \frac{7.5}{6} = \frac{5}{4} = 125\%$. Thus, $n = \mathbf{125}$.

6) Use the theorem that the line joining the midpoints of two sides of a triangle is parallel to the third side and its length is half the length of the third side. Since $\overline{MN} \parallel \overline{QR}$, $\measuredangle MTQ \cong \measuredangle TQR$. Since $\overrightarrow{QS}$ is an angle bisector, $\measuredangle MQT \cong \measuredangle TQR$. Therefore, $MQ = MT = 2$, and
$MN = \frac{1}{2}QR = 5$. So, $TN = 3$, the required ratio is 2/3 and the required sum is **5**.