Nassau County Interscholastic Mathematics League

Contest #4 Answers must be integers from 0 to 999 inclusive. 2013 – 2014

Calculators are allowed.

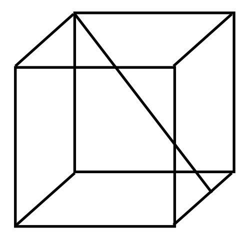
Time: 10 minutes

1. What is the hundredth digit to the right of the decimal point in the decimal representation of $\frac{2}{7}$?

2. The vertices of a triangle are $(1,7), (5,-2)$, and $(0,1)$ Find the sum of the coordinates of the centroid of the triangle.

Time: 10 minute

3. Allen's room is a cube whose edges are each 12 feet. As he fell asleep, he dreamt of fireflies that fly in a perfectly straight line from one corner on the ceiling to the midpoint of an edge on the floor as shown in the diagram.
Compute the distance traveled by one of these fireflies.



4. The length of hypotenuse $\overline{AB}$ of a right triangle ABC is 41 inches and the radius of the inscribed circle O is 4 inches. Find the number of inches in the perimeter of right triangle ABC.

Time: 10 minutes

5. For all real $x, 8f(x) + 2f(20-x) = x^3$. Compute $f(10)$.

6. If $0 \leq \theta \leq \frac{\pi}{2}$ and $\sin\theta + \cos\theta = \frac{2}{\sqrt{3}}$, compute $\tan\theta + \cot\theta$.

Solutions for Contest #4

1.**7.** $\frac{2}{7} = 0.\overline{285714}$ Since the repeating part contains six digits, the ninety-sixth digit is a 4 and the hundredth digit is a 7.

2. **4.** The coordinates of a centroid of a triangle can be determined by averaging the *x*-coordinates and then averaging the *y*-coordinates of the vertices. So, the coordinates of the centroid are $\left(\frac{1+5+0}{3}, \frac{7-2+1}{3}\right) = (2,2)$ and the required sum is 4.

3. **18.** We can use the Pythagorean Theorem in three dimensions. The required distance is $\sqrt{12^2 + 12^2 + 6^2} = \sqrt{324} = 18.$

4. **90.** The points of tangency on $\overline{AB}, \overline{BC}$, and $\overline{AC}$ are R, S, and T respectively. Quadrilateral $OTCS$ is a square. Let $AT = AR = x$ and $BR = BS = 41 - x$. The perimeter of right triangle $BC = AB + BS + CS + TC + AT = 41 + 41 - x + 4 + 4 + x = 90.$

Method 2: We might guess that this could be a 9-40-41 right triangle. To verify that this is the case, use the area formula *A* = *rs*, where *r* is the inradius and *s* is the semi-perimeter, $s = \frac{9+40+41}{2} = 45$. Solving for the radius, we have
$r = \frac{A}{s} = \frac{180}{45} = 4$ as the problem stated. The perimeter is 9 + 40 + 41 = 90.

5. **100.** Substitute $x = 10$ into the given equation to get $8f(10) + 2f(10) = 10^3$ or $10f(10) = 1000$ and $f(10) = 100$.

6. **6.** Note that $\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta} = \frac{1}{\cos\theta\sin\theta}$.

Also, $(\sin\theta + \cos\theta)^2 = \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta = 1 + 2\sin\theta\cos\theta = \frac{4}{3}$.

Thus, $\sin\theta\cos\theta = \frac{1}{6}$ and $\frac{1}{\cos\theta\sin\theta} = 6.$

Method 2: We can solve $\sin\theta + \cos\theta = \frac{2}{\sqrt{3}}$ with a graphing calculator and get θ≈0.169918 or θ ≈1.40088.

For either value of θ, $\tan\theta + \cot\theta = 6$.