Nassau County Interscholastic Mathematics League

Contest #1 Answers must be integers from 0 to 999 inclusive. 2013 – 2014

No calculators are allowed.

Time: 10 minutes

1. What is the largest three-digit number divisible by 3, 4, 5, and 6?

2. A sector of a circle with a central angle whose measure is 60° has an arc whose length is 10π. If the area of the sector is $k\pi$, compute k.

Time: 10 minutes

3. Let *A* = [(2013 +6)(2013 – 6) + (2013 – 6) (2013 +6)] and
B = [(2013 + 6)(2013 – 6) – (2013 – 6) (2013 +6)]. Compute the product of A and B.

4. What is the largest number of cards that can be randomly drawn from a standard deck of 52 cards before six cards of any particular suit are drawn?

Time: 10 minutes

5. If the sum of two numbers is 7 and the sum of their multiplicative inverses is $\frac{7}{10}$, find their product.

6. The function f is defined recursively as $f(1) = 5$ and $f(n+1) = \frac{2f(n)+1}{2}$ for $n = 1, 2, 3, \cdots$.

If $f(n+1) = 1012$, compute the remainder when n is divided by 1000.

Solutions for Contest #1

1. **960**. The least common multiple of 3, 4, 5, and 6 is 60. The largest three-digit number divisible by 60 is 960.

2. **150**. The length of the arc of the sector is one-sixth the circumference of the circle. So, $10\pi = \frac{1}{6} \cdot 2\pi r$. Therefore, $r = 30$ and the area of the sector is $\frac{1}{6}\pi 30^2 = 150\pi$ and $k = 150$.

3. **0.** The second number in the product is zero, so the product is 0.

4. **20.** Since one might draw five cards of each of four suits, the largest number of cards that can be randomly drawn from a standard deck of cards is 20.

5. **10.** If the numbers are x and y, then $\frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} = \frac{7}{xy} = \frac{7}{10}$, and $xy = 10$.

6. **14**. We notice that $f(2) = \frac{11}{2}, f(3) = \frac{12}{2}, f(4) = \frac{13}{2}, \cdots$ and conclude that explicitly $f(n) = \frac{n+9}{2}$. Since $f(n+1) = 1012$, we solve $\frac{n+10}{2} = 1012$ to get $n = 2014$. The required remainder is 14.