Nassau County Interscholastic Mathematics League
Contest #1 Answers must be integers from 0 to 999 inclusive. 2012 – 2013
No calculators are allowed.

**Time: 10 minutes**

1. If 170% of 2.8 is 1.4% of $x$, compute $x$.

2. The base of a pyramid is a square with sides of length 2. The other four edges of the pyramid have length $\sqrt{38}$. Compute the volume of the pyramid.

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**Time: 10 minutes**

3. The first three terms of an arithmetic sequence are log 5, log $x$, and log 45, respectively. Compute $x$.

4. The lengths of the three sides of right triangle form a geometric sequence, and the shortest side has length 6. Given that the length of the hypotenuse is $m+\sqrt{n}$, where $m$ and $n$ are positive integers, find $m + n$.

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**Time: 10 minutes**

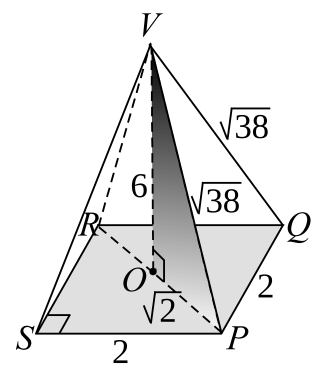
5. An 8 by 8 magic square is a grid of 64 unit squares arranged to form an 8 by 8 square. Each of the unit squares contains an integer between 1 and 64 inclusive, and the sums of each row, each column, and each long diagonal are equal. Compute that sum.

6. Find the sum of all values of $x$ between 0 and $2\pi$ that satisfy $12\sin^2 x = \sin x + 1$. Round your answer to the nearest integer.

# Solutions for Contest #1

1. The given condition is equivalent to 1.7(2.8) = 0.014$x$. Solve to obtain $x$ = 340.

2. The volume of a pyramid is given by $V = (1/3)Bh$, where $B$ is the area of the base of the pyramid and $h$ is the height of the pyramid. In this case $B = 4$.
Let $PQRS$ be the base of the pyramid, and let $V$ be its other vertex. Let $O$ be the center of the base. To find $h$, consider right triangle $VOP$.
Because $OP = 1/2(PR) = 1/2\left(2\sqrt{2}\right) = \sqrt{2}$, conclude that $h = \sqrt{\left(\sqrt{38}\right)^2 - \left(\sqrt{2}\right)^2}$, so $h = 6$. Thus, the volume of the pyramid is $\frac{1}{3} \cdot 4 \cdot 6 = 8$.




3. In the arithmetic sequence: $a, b, c,$ it is true that $b - a = c - b$.
Therefore, $\log x - \log 5 = \log 45 - \log x \rightarrow \log\left(\frac{x}{5}\right) = \log\left(\frac{45}{x}\right) \rightarrow \frac{x}{5} = \frac{45}{x} \rightarrow$ $x^2 = 225 \rightarrow x = 15$.

4. Let $r$ be the ratio of the geometric sequence. Then $6^2 + (6r)^2 = (6r^2)^2$, and so $r^4 - r^2 - 1 = 0$. Use the quadratic formula to find that $r^2 = \frac{1+\sqrt{5}}{2}$. Hence, the hypotenuse equals $6r^2 = 3 + 3\sqrt{5} = 3 + \sqrt{45}$, so $m + n$ = 48.

5. The sum of the 64 integers in the magic square is 64(64 + 1)/2 = 32 · 65. The sum of each row is 1/8 of that, namely 4 · 65 = 260, so the requested sum is 260.

6. The given equation is equivalent to $(3\sin x - 1)(4\sin x + 1) = 0$, so $\sin x = \frac{1}{3}$ or $\sin x = -\frac{1}{4}$. Let $\theta_1 = \sin^{-1}\left(\frac{1}{3}\right)$. Then the two solutions corresponding to $\sin x = \frac{1}{3}$ are $\theta_1$ and $\pi - \theta_1$. Let $\theta_2 = \sin^{-1}\left(-\frac{1}{4}\right)$. Note that $\theta_2$ is negative. So, the two solutions corresponding to $\sin x = -\frac{1}{4}$ are $\pi - \theta_2$ and $2\pi + \theta_2$. The sum of the four solutions is thus $4\pi$, and this rounds to 13.

Nassau County Interscholastic Mathematics League
Contest #2 Answers must be integers from 0 to 999 inclusive. 2012 – 2013
Calculators are allowed.

**Time: 10 minutes**

1. In quadrilateral $ABCD$, $\overline{AB} \perp \overline{BC}$, diagonal $\overline{AC} \perp \overline{CD}$, $AB = 9$, $BC = 3\sqrt{7}$, and $CD = 5$. Compute $AD$.

2. A student wishes to divide a group of eight teachers into two groups of four. How many such divisions are possible?

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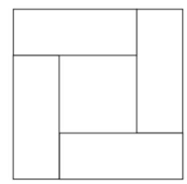
**Time: 10 minutes**

3. The diagram (which is not drawn to scale) shows four congruent rectangles arranged so that they determine two squares. The areas of the squares are 25 and 121. Find the area of one of the rectangles.



4. Let $r$ be the positive root of $x^2 = x + 1$. Given that $r^3 = m + \sqrt{n}$, where $m$ and $n$ are positive integers, compute $m + n$.

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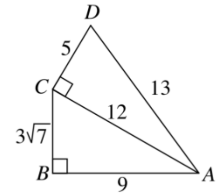
**Time: 10 minutes**

5. Compute the number of positive integer divisors of $10^9$ that do not end in 0.

6. In quadrilateral $ABCD, AB = 6, BC = 7, CD = 8\sqrt{3}$, $m\measuredangle B = 90°$ and $m\measuredangle C = 120°$ Given that $AD^2 = m + n\sqrt{p}$, where $m$, $n$, and $p$ are positive integers and $p$ is not divisible by the square of a prime, find $m + n + p$.

## Solutions for Contest #2




1. Use the Pythagorean Theorem in $\Delta ABC$ to conclude that $AC = 12$, then use it again in $\Delta ACD$ to find that $AD = 13$.

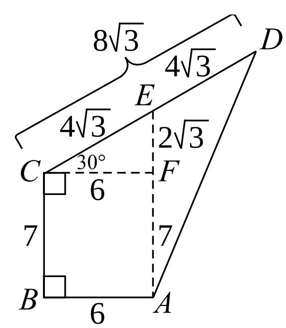
2. There are $\binom{8}{4} = 70$ four-element subsets of the set of eight teachers. Thus, there are $\frac{70}{2} = 35$ of the desired divisions because each subset and its complement correspond to the same division.

3. Let the dimensions of one of the rectangles be $a$ and $b$ with $a > b$. Then $a + b = 11$ and $a - b = 5$. Solve the system to find that $a = 8$ and $b = 3$. The requested area is therefore 24.

4. Use the Quadratic Formula to find that $r = \frac{1+\sqrt{5}}{2}$. The given equation implies that $r^2 = r + 1$. Multiply by $r$ to obtain $r^3 = r^2 + r = r + 1 + r = 2r + 1 = 2 + \sqrt{5}$. Thus, $m + n = 7$.
   Alternate solution: Let $r = \frac{1+\sqrt{5}}{2} \rightarrow r^2 = \frac{3+\sqrt{5}}{2} \rightarrow r^3 = r^2 \cdot r = 2 + \sqrt{5}$. Again, $m + n = 7$.

5. The prime factors of a divisor of $10^9$ must consist of 2's and/or 5's, so the divisors that do not end in 0 must have only factors of 2 or only factors of 5. That is, they must be of the form $2^a$ or $5^a$, where $0 \le a \le 9$. Because $2^0 = 5^0 = 1$, there are $10 + 10 - 1 = 19$ of the desired divisors.

6. Draw the line through $A$ that is parallel to $\overline{BC}$, and let $E$ be the point where the line intersects $\overline{CD}$. Let $F$ be the projection of $C$ onto $\overline{AE}$. Then $ABCF$ is a rectangle and $\Delta CEF$ is $30° - 60° - 90°$, so $AF = 7, CF = 6, FE = 2\sqrt{3}$, and $CE = 4\sqrt{3}$. Hence, $ED = 4\sqrt{3}$. Apply the Law of Cosines in $\Delta AED$ to find that $AD^2 = 133 + 56\sqrt{3}$. Thus, $m + n + p = 192$.



Nassau County Interscholastic Mathematics League
Contest #3 Answers must be integers from 0 to 999 inclusive. 2012 – 2013
No calculators are allowed.

**Time: 10 minutes**

1. A rectangular field measures 60 yards by 80 yards. Farrel starts at one corner of the field and heads directly to the opposite corner at an average rate of 35 yards per minute. Kevin starts at the same corner and the same time as Farrel does, but he travels to the opposite corner along the edges of the field. Compute the number of yards per minute Kevin must average to arrive at the opposite corner at the same time Farrel does.

2. For how many real numbers $x$ is it true that $(x+3)^8 = (x^2-9)^4$?

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**Time: 10 minutes**

3. In $\Delta ABC$, $m\measuredangle A = 54°, m\measuredangle B = 59°$ and point $O$ is the center of the circumscribed circle. Compute $m\measuredangle OAC$ in degrees.

4. If $\tan\left(\frac{\pi}{4} - x\right) + \cot\left(\frac{\pi}{4} - x\right) = 4$, compute $\cot^2 x$.

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**Time: 10 minutes**

5. For how many positive integers $n$ does 100 leave a remainder of 4 when 100 is divided by $n$?

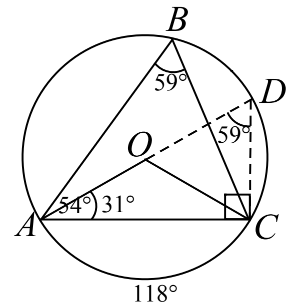
6. If $\log_x 4 = \log_{3x} 324$, then find $x^2$.

## Solutions for Contest #3

1. Apply the Pythagorean Theorem to find that Farrel must travel 100 yards. Because the ratio of Kevin's distance to Farrel's distance is 140/100 = 7/5, the ratio of Kevin's average velocity to Farrel's must also be 7/5. Thus, Kevin's average velocity must be 7/5 of 35 or 49 yards per minute.

2. The given equation is equivalent to $(x+3)^8 = (x+3)^4(x-3)^4$, which is satisfied when $x+3=0$, that is, when $x=-3$. If $x \neq -3$, divide both sides by $(x+3)^4$ to get $(x+3)^4 = (x-3)^4$, which is satisfied if and only if $x+3 = -(x-3)$, that is, when $x=0$. Thus, there are two solutions to the given equation, namely 0 and $-3$.

3. Extend $\overline{AO}$ to form diameter $\overline{AD}$. Then $\measuredangle ADC$ and $\measuredangle B$ intercept the same arc (arc $AC$), so $m\measuredangle ADC = m\measuredangle B = 59°$. Also, $\measuredangle ACD$ is a right angle because it is inscribed in a semicircle. Thus, $m\measuredangle OAC = 90° - 59° = 31°$.
<u>Alternate solution</u>: Draw congruent radii $\overline{OA}$ and $\overline{OC}$. Since $m\measuredangle B = 59°, m$ (arc $AC$) $= m\measuredangle AOC = 118°$. So, $m\measuredangle OAC = \frac{1}{2}(180° - 118°) = 31°$.




4. Use the formula for the tangent of a difference of two angles to get $\tan\left(\frac{\pi}{4} - x\right) = \frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x} = \frac{1-\tan x}{1+\tan x}$, so $\cot\left(\frac{\pi}{4} - x\right) = \frac{1+\tan x}{1-\tan x}$.
Thus, the given equation is equivalent to $\frac{1-\tan x}{1+\tan x} + \frac{1+\tan x}{1-\tan x} = 4$.

Clear fractions to obtain $2 + 2(\tan x)^2 = 4 - 4(\tan x)^2$ .

Then $(\tan x)^2 = \frac{1}{3}$. So $(\cot x)^2 = 3$.

5. The given conditions imply that $n > 4$ and that 100 is 4 more than a multiple of $n$, that is, 96 is a multiple of $n$. But 96 has 12 divisors, and 8 of them are greater than 4. So, there are 8 of the desired integers.

6. Let $y = \log_x 4 = \log_{3x} 324$. Then $4 = x^y$ and $324 = (3x)^y = 3^y \cdot x^y$. Divide the equations to get $324/4 = 81 = 3^y$. So, $y = 4$. Hence, $x^4 = 4$, and $x^2 = 2$.

Nassau County Interscholastic Mathematics League

Contest #4 Answers must be integers from 0 to 999 inclusive. 2012 – 2013

Calculators are allowed.

**Time: 10 minutes**

1. If $10 \leq n \leq 99$, find the integer $n$ such that $n$ divided by the sum of its digits is a minimum.

2. Quadrilateral $ABCD$ is cyclic (that is, it has a circumscribed circle). If $AB = AD = 5$, and $CB = CD = 10$, compute $BD^2$.

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**Time: 10 minutes**

3. In Priscilla's Lottery, you and Priscilla each pick 5 distinct positive integers between 1 and 50, inclusive. You win if your five numbers match the ones that are picked by Priscilla. In Quincy's Lottery, you and Quincy each pick 5 distinct positive integers between 1 and 50, inclusive and then randomly designate one as special. You win if your special integer is the same as Quincy's and your other four integers are the same as Quincy's other four. Let $p$ be the probability that you win Priscilla's Lottery and let $q$ be the probability that you win Quincy's Lottery. Compute $p/q$.

4. When ice melts to water, the volume of the water is 90% of the volume of the ice. A glass containing ice and water is filled to the top. When all the ice melts, the glass is 96% full. What per cent of the original volume was ice?

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**Time: 10 minutes**

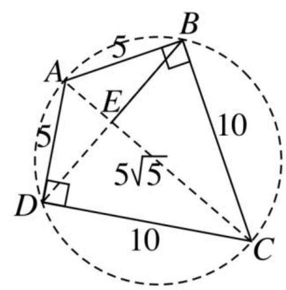
5. In equilateral triangle $ABC$ with $AB = 8$, points $P$ and $Q$ are chosen on side $\overline{AB}$ so that $AP = BQ = 2$. Similarly, points $R$ and $S$ are chosen on side $\overline{BC}$ so that $BR = CS = 2$, and points $T$ and $U$ are chosen on side $\overline{CA}$ so that $CT = AU = 2$. If the area of hexagon $PQRSTU = H$, find $H^2$.

6. Given that $\sum_{k=1}^{89} \sin^2 k^\circ = m/n$, where $m$ and $n$ are relatively prime positive integers, find $m + n$.

## Solutions for Contest #4

1. Represent the tens' digit and units' digit of a two-digit number by $a$ and $b$, respectively, and denote the desired quotient by $Q$.
Then $Q = (10a + b)/(a + b) = 1 + (9a)/(a + b)$. Thus, $Q$ is minimum when $b = 9$.
When $b = 9$, $Q - 1 = (9a)/(a + 9) = 9/(1 + 9/a)$. Thus, $Q$ is minimum
when $1 + 9/a$ is maximum or when $a = 1$. Therefore, the minimum value of $Q$ is $19/(1 + 9) = 19/10$ and $n = 19$.
Alternate solution: $\frac{10a+b}{a+b} = \frac{10a+10b}{a+b} - \frac{9b}{a+b} = 10 - \frac{9b}{a+b}$ is a minimum when $a = 1$ and $b = 9 \rightarrow n = 19$.

2. Notice that $\overleftrightarrow{AC}$ is a symmetry line for the quadrilateral and its circumscribed circle. Therefore, $\overline{AC}$ is a diameter of the circle.
Use the Pythagorean Theorem in $\Delta ABC$ to find that $AC = 5\sqrt{5}$.
The symmetry implies that $BD = 2 \cdot BE$, where point $E$ is the foot of an altitude of $\Delta ABC$ from vertex $B$. Because $\overline{BE}$ is a height of $\Delta ABC$, conclude that $5 \cdot 10 = 2|ABC| = 5\sqrt{5}.\cdot BE$.
Thus, $BE = 2\sqrt{5}.$, and so $BD = 4\sqrt{5} = \sqrt{80}$. Thus, $BD^2 = 80$.

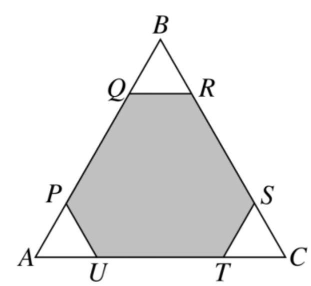



Alternate Solution 1: Use the Law of Cosines in $\Delta BAD$ and in $\Delta BCD$. Note that $\cos\measuredangle BCD = -\cos\measuredangle BAD$.
So, $BD^2 = 50 - 50\cos\measuredangle BAD$ and $BD^2 = 200 + 200\cos\measuredangle BAD \rightarrow BD^2 = 80$.
Alternate Solution 2: Use Ptolemy's Theorem: $AC \cdot BD = AB \cdot CD + AD \cdot BC$ together with the fact that $AC = 5\sqrt{5}$. from the first solution.

3. Since $p = \frac{1}{\binom{50}{5}}$ and $= \frac{1}{5} \cdot \frac{1}{\binom{50}{5}}$, $\frac{p}{q} = 5$.

4. Let $I$ and $W$ be the fractions of the volume of the glass that were ice and water, respectively, originally. Then $I + W = 1$. Let $I'$and $W'$ be the fractions of the volume of the glass that are ice and water, respectively, after the ice melts. Then $I' + W' = 96$. But $I' = 0$, and $W' = W + .9I$. Thus, $W + .9I = .96$. Subtract the last equation from the first to find that $.1I = .04$, so $I = .4$. Hence, 40% of the original volume was ice.

5. Triangles $APU$, $BQR$, and $CST$ are equilateral, and the area of each of them is 1/16 that of $\Delta ABC$. Thus, $H$ is 13/16 the area of $\Delta ABC$. Use the formula $K = (s^2\sqrt{3}) / 4$ to find that the area of $\Delta ABC$ is $16\sqrt{3}$. Then $H = 13\sqrt{3}$, so $H^2 = 507$.




Alternate solution: Draw $\overline{RU}$. Using $30° - 60° - 90°$ triangles, we can calculate the height of trapezoid $URQP$ to be $\sqrt{3}$ and the height of trapezoid $TSRU$ to be $2\sqrt{3}$. The required area is the sum of the areas of the trapezoids: $\frac{1}{2} \cdot (4 + 6) \cdot \sqrt{3} + \frac{1}{2} \cdot (2 + 6) \cdot 2\sqrt{3} = 13\sqrt{3}$.

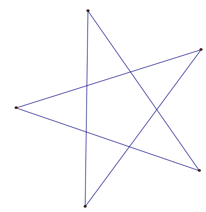
6. Let $S=((\sin 1°)^2+(\sin 2°)^2+\cdots+(\sin 43°)^2+(\sin 44°)^2)+(\sin 45°)^2+$
$((\sin 46°)^2+(\sin 47°)^2+\cdots+(\sin 89°)^2)$. $\rightarrow$
$S=((\sin 1°)^2+(\sin 2°)^2+\cdots+(\sin 43°)^2+(\sin 44°)^2)+(\sin 45°)^2+$
$((\cos 44°)^2+(\cos 43°)^2+\cdots+(\cos 1°)^2)$ . Re-arrange to get
$S=((\sin 1°)^2+(\cos 1°)^2)+((\sin 2°)^2+(\cos 2°)^2)+\cdots$
$+((\sin 44°)^2+(\cos 44°)^2)+(\sin 45°)^2=44+\frac{1}{2}=\frac{89}{2}$. So, $m+n=91$.

Nassau County Interscholastic Mathematics League
Contest #5 Answers must be integers from 0 to 999 inclusive. 2012 – 2013
No calculators are allowed.

**Time: 10 minutes**

1. The units digit of the product of four consecutive positive integers is $c$. Compute the number of possible values of $c$.

2. In the diagram, the regular five-pointed star consists of five congruent segments, and the five angles at its vertices are congruent. Compute the degree-measure of one of those angles.



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**Time: 10 minutes**

3. If $n^2 = \frac{28! + 29! + 30!}{28!}$, what is the positive value of $n$?

4. Find the only three-digit prime factor of $4^{12} - 3^{12}$.

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**Time: 10 minutes**

5. The difference of the roots of $x^2 + bx + 306 = 0$ is 1, and $b > 0$. Compute $b$.

6. Find the sum of all values of $x$ between $0°$ and $180°$ that satisfy $\cos 3x + \cos 2x + \cos x = 0$, where all angles are measured in degrees.

## Solutions for Contest #5

1. Notice that $c$ is determined by the last digits of the four integers. For any such set of four last digits that contains a 0 or a 5, $c$ must equal 0. The only sets that do not contain a 0 or a 5 are {1, 2, 3, 4} and {6, 7, 8, 9}, and in both of these cases, $c = 4$. Thus $c$ has two possible values.

2. Circumscribe a circle about the star. The star divides the circle into five congruent arcs, so the measure of each of those arcs must be 72°. Since each of the star's angles is an inscribed angle that intercepts one of the arcs, the measure of each angle is 36°.

3. Re-write the given: $n^2 = \frac{28!(1+29+29\cdot 30)}{28!} = 1 + 29 + 870 = 900$, so $n = 30$.

4. Factor $4^{12} - 3^{12} = (4^6 - 3^6)(4^6 + 3^6) = (16^3 - 9^3)(16^3 + 9^3)$
$= (16 - 9)(16^2 + 16 \cdot 9 + 9^2)(16 + 9)(16^2 - 16 \cdot 9 + 9^2) = 7 \cdot 481 \cdot 25 \cdot 193$.
Check to see that 193 is a prime. The statement of the problem implies that it must be the only three-digit prime factor. (Note that $481 = 13 \cdot 37$.)

5. The positive difference of the roots of $x^2 + bx + c = 0$ is
$\frac{-b+\sqrt{b^2-4c}}{2} - \frac{-b-\sqrt{b^2-4c}}{2} = \sqrt{b^2 - 4c}$. The given conditions therefore imply that
$b^2 - 4 \cdot 306 = 1$, and so $b^2 = 1225$. Thus, $b = 35$.
Alternate solution: The roots are $r$ and $s$. The given condition implies that $r - s = 1$. Square this equation to get $r^2 + s^2 - 2rs = 1$. Use the sum and product of the roots to get $rs = 306$ and $r + s = -b$.
So, $r^2 + s^2 = 613$ and $r^2 + s^2 + 2rs = 1225 \ \rightarrow |r + s| = 35$.
Since $b > 0, b = 35$.

6. Use the formulas for the cosine of a sum and a difference to get
$\cos 3x = \cos(2x + x) = \cos 2x \cos x - \sin 2x \sin x$, and that
$\cos x = \cos(2x - x) = \cos 2x \cos x + \sin 2x \sin x$. Add to get
$\cos 3x + \cos x = 2\cos 2x \cos x$ for all $x$, so the given equation is equivalent to $2\cos 2x \cos x + \cos 2x = 0$. Factor to obtain
$(\cos 2x)(2\cos x + 1) = 0$ Then $\cos 2x = 0$ or $\cos x = -\frac{1}{2}$. The degree measures of the requested solutions are therefore 45, 135, and 120, and their sum is 300. Alternate solution: Again use the cosine of a sum to get
$\cos 2x \cos x - \sin 2x \sin x + \cos 2x + \cos x = 0$
$\cos 2x \cos x - 2(\sin x)^2 \cos x + \cos 2x + \cos x = 0$
$\cos 2x \cos x - 2\cos x(1 - (\cos x)^2) + \cos 2x + \cos x = 0$
$\cos 2x \cos x - 2\cos x + 2(\cos x)^3 + \cos 2x + \cos x = 0$
$2(\cos x)^3 - \cos x - 2\cos x + 2(\cos x)^3 + 2(\cos x)^2 - 1 + \cos x = 0$
$4(\cos x)^3 + 2(\cos x)^2 - 2\cos x - 1 = 0$
$2(\cos x)^2(2\cos x + 1) - (2\cos x + 1) = 0$
$(2(\cos x)^2 - 1)(2\cos x + 1) = 0$ Finish as above.

Nassau County Interscholastic Mathematics League

Team Contest    Answers must be integers from 0 to 999 inclusive.    2012 – 2013

Calculators are allowed.

1. Find the smallest positive integer that is not a divisor of 31!

2. Find the sum of the integer solutions of $1 < (x-5)^2 < 100$.

3. Compute the minimum possible sum of digits for a positive integer that is a multiple of 17.

4. One root of $x^2 + kx - 4 = 0$ is the square of the other root. Find the sum of the cubes of the roots.

5. A tetromino is a figure that consists of four unit squares, each of which shares at least one side with at least one of the other three squares. Compute the number of non-congruent tetrominos.

6. If $\sin x - \cos x = \frac{1}{2}$, and $\sin 2x = \frac{m}{n}$, where $m$ and $n$ are relatively prime positive integers, compute $m + n$.

7. A 2 by 3 rectangle is to be covered by 1 by 2 rectangles and 1 by 1 squares. The 1 by 2 rectangles and 1 by 1 squares must not overlap, and their sides must be parallel to the sides of the 2 by 3 rectangle. Compute the number of possible patterns for such coverings.

8. If $\frac{1}{\sqrt[3]{2}-1} = a + \sqrt[3]{b} + \sqrt[3]{c}$, and $a$, $b$, and $c$ are positive integers, **with $a < b < c$**, find $100a + 10b + c$.

9. The centers of circles $O$ and $P$ are inside equilateral triangle $ABC$, and their radii are 1 and 2, respectively. Circles $O$ and $P$ are externally tangent, circle $O$ is tangent to $\overline{AB}$ and $\overline{AC}$, and circle $P$ is tangent to $\overline{AB}$ and $\overline{BC}$. Given that $AB = \sqrt{m} + \sqrt{n}$, where $m$ and $n$ are positive integers, find $m + n$.

10. Find the value of $x$ for which: $\left(\frac{1+\sqrt{5}}{2}\right)^{2012} + \left(\frac{1+\sqrt{5}}{2}\right)^{2013} = \left(\frac{1+\sqrt{5}}{2}\right)^{x}$.

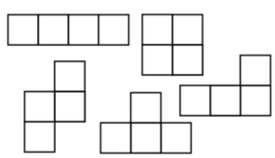
## Solutions for Team Contest

1. The positive integers from 1 to 31 inclusive all divide 31!. So do $32, 33, 34, 35$, and 36 because their prime factors are less than 31. But 37 does not divide 31! because 37 is not a prime factor of any of the integers from 1 to 31.

2. Use a transformation. Each of the solutions of $1 < (x-5)^2 < 100$ is 5 greater than a corresponding solution of $1 < x^2 < 100$. There are 16 solutions of the latter inequality, and their sum is 0. So, the sum of the solutions of the original inequality is $16 \cdot 5 = 80$.
Alternate solution: Since $\sqrt{a^2} = |a|$ for all values of $a$, the original inequality is equivalent to $1 < |x-5| < 10$. Thus, the distance on the number line between the point whose coordinate is 5 and the point whose coordinate is represented by $x$ is greater than one and less than ten. The integers that meet these requirements satisfy $(x < 4$ or $x > 6)$ and $-5 < x < 15$. The sum of these sixteen integers is 80.

3. A number whose digit-sum is 1 must consist of a 1 followed by any number of 0's (including none), but no number ending in 0 whose digit-sum is 1 can be a multiple of 17. If a number whose digit-sum is 2 were to be a multiple of 17, it would have to consist of two 1's with some number of 0's between them. Check on a calculator to find that 100000001 is a multiple of 17. Thus, the minimum digit-sum is 2.
Alternate solution: Note that $10^2 = 100 \equiv 15 \pmod{17} \equiv -2 \pmod{17}$ .
So, $10^8 = \left(10^2\right)^4 \equiv (-2)^4 \pmod{17} \equiv 16 \pmod{17}$. Therefore,
$10^8 + 1$ is a multiple of 17.

4. Denote the roots by $r$ and $r^2$. Then $-4 = r \cdot r^2$, so $r = \sqrt[3]{-4}$, and so $r^2 = \sqrt[3]{16}$. Thus, the requested sum is $-4 + 16 = 12$.

5. There are five. 

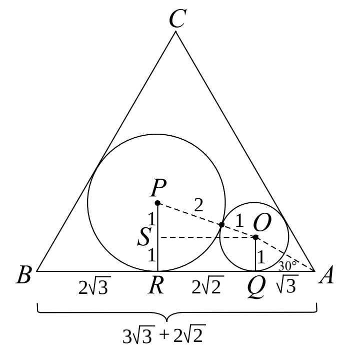
6. Square both sides to obtain $\sin^2 x + \cos^2 x - 2\sin x \cos x = ¼$. This is equivalent to $1 - \sin 2x = ¼$, so $\sin 2x = ¾$ and $m + n = 7$.

7. The number of 1 by 2 rectangles can be 0, 1, 2, or 3. Once all of them are placed, there is only one way to place the 1 by 1 squares. When there are 0 such rectangles, there is 1 possible pattern. There are 7 possible placements for 1 such rectangle because there are 2 placements in each of the two rows and 1 in each if the 3 columns. To place 2 rectangles, consider 3 cases: both horizontal (HH),

both vertical (VV), and one horizontal and one vertical (HV). There are 4 placements of the first type, 3 of the second, and 4 of the third for a total of 11. For 3 rectangles, the cases are HHH, HHV, HVV, and VVV, and there are 0, 2, 0, and 1 placements, respectively, for these cases for a total of 3. Thu,s there are 1 + 7 + 11 + 3 = 22 possible patterns.

8. Let $x = \sqrt[3]{2}$.
Then $\frac{1}{\sqrt[3]{2}-1} = \frac{1}{x-1} = \frac{x^2+x+1}{(x-1)(x^2+x+1)} = \frac{x^2+x+1}{x^3-1} = x^2 + x + 1 = \sqrt[3]{4} + \sqrt[3]{2} + 1$.
Since $a < b < c, a = 1, b = 2,$ and $c = 4$. Thus, $100a + 10b + c = 124$.

9. Let $Q$ and $R$ be the projections of $O$ and $P$, respectively, onto $\overline{AB}$. Notice that $\Delta OAQ$ is a $30° - 60° - 90°$ triangle, so $AQ = \sqrt{3}$. Similarly, $BR = 2\sqrt{3}$. In quadrilateral $OPRQ$, $OP = 1 + 2 = 3$, $PR = 2$, and $OQ = 1$. Let $S$ be the projection of point $O$ onto $\overline{PR}$. Then $PS = 2 – 1 = 1$.
So, $R = OS = \sqrt{3^2 - 1} = 2\sqrt{2}$ .
Thus, $AB = 3\sqrt{3} + 2\sqrt{2} = \sqrt{27} + \sqrt{8}$. So, $m + n = 35$.




10. Let $p = \frac{1+\sqrt{5}}{2}$. Then, $p^{2012}(1 + p) = p^x$ or $1 + p = p^{x-2012}$.
So $\frac{3+\sqrt{5}}{2} = \left(\frac{1+\sqrt{5}}{2}\right)^{x-2012}$ . Therefore, $\frac{3+\sqrt{5}}{2}$ is a power of $\frac{1+\sqrt{5}}{2}$.
So, $x - 2012 = 2$ and $x = 2014$.
<u>Alternate solution</u>: Let $p = \frac{1+\sqrt{5}}{2}$, and let $q = \frac{1-\sqrt{5}}{2}$.
Then $p + q = 1$ and $pq = -1$. So $p$ and $q$ are roots of $x^2 - x - 1 = 0$.

Thus, $p$ must satisfy $p^2 = p + 1$. Multiply both sides by $p^{2012}$ to find

that $p^{2014} = p^{2013} + p^{2012}$, and that therefore $x = 2014$ is a solution of the given equation. The given equation must have a unique solution because $f(x) = p^x$ is an increasing function whose range is the set of positive real numbers.