Time: 10 minutes

1. If 170% of 2.8 is 1.4% of x, compute x.

2. The base of a pyramid is a square with sides of length 2. The other four edges of the pyramid have length $\sqrt{38}$. Compute the volume of the pyramid.

Time: 10 minutes

3. The first three terms of an arithmetic sequence are log 5, log x, and log 45, respectively. Compute x.

4. The lengths of the three sides of right triangle form a geometric sequence, and the shortest side has length 6. Given that the length of the hypotenuse is $m+\sqrt{n}$, where m and n are positive integers, find $m + n$.

Time: 10 minutes

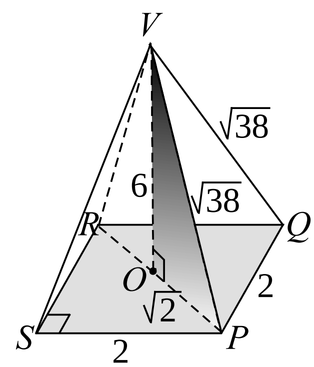
5. An 8 by 8 magic square is a grid of 64 unit squares arranged to form an 8 by 8 square. Each of the unit squares contains an integer between 1 and 64 inclusive, and the sums of each row, each column, and each long diagonal are equal. Compute that sum.

6. Find the sum of all values of x between 0 and 2π that satisfy $12 \sin^2 x = \sin x + 1$. Round your answer to the nearest integer.

Solutions for Contest #1

1. The given condition is equivalent to $1.7(2.8) = 0.014x$. Solve to obtain $x = 340$.

2. The volume of a pyramid is given by $V = (1/3)Bh$, where B is the area of the base of the pyramid and h is the height of the pyramid. In this case $B = 4$.
Let $PQRS$ be the base of the pyramid, and let V be its other vertex. Let O be the center of the base. To find h, consider right triangle VOP.
Because $OP = 1/2(PR) = 1/2(2\sqrt{2}) = \sqrt{2}$, conclude that $h = \sqrt{(\sqrt{38})^2 - (\sqrt{2})^2}$, so $h = 6$. Thus, the volume of the pyramid is $\frac{1}{3} \cdot 4 \cdot 6 = 8$.



3. In the arithmetic sequence: $a, b, c,$ it is true that $b - a = c - b$.
Therefore, $\log x - \log 5 = \log 45 - \log x \rightarrow \log\left(\frac{x}{5}\right) = \log\left(\frac{45}{x}\right) \rightarrow \frac{x}{5} = \frac{45}{x} \rightarrow$
$x^2 = 225 \rightarrow x = 15$.

4. Let r be the ratio of the geometric sequence. Then $6^2 + (6r)^2 = (6r^2)^2$, and so $r^4 - r^2 - 1 = 0$. Use the quadratic formula to find that $r^2 = \dfrac{1+\sqrt{5}}{2}$. Hence, the hypotenuse equals $6r^2 = 3 + 3\sqrt{5} = 3 + \sqrt{45}$, so $m + n = 48$.

5. The sum of the 64 integers in the magic square is $64(64 + 1)/2 = 32 \cdot 65$. The sum of each row is 1/8 of that, namely $4 \cdot 65 = 260$, so the requested sum is 260.

6. The given equation is equivalent to $(3\sin x - 1)(4\sin x + 1) = 0$, so $\sin x = \frac{1}{3}$ or $\sin x = -\frac{1}{4}$.
Let $\theta_1 = \sin^{-1}\left(\frac{1}{3}\right)$. Then the two solutions corresponding to $\sin x = \frac{1}{3}$ are θ_1 and $\pi - \theta_1$. Let $\theta_2 = \sin^{-1}\left(-\frac{1}{4}\right)$. Note that θ_2 is negative. So, the two solutions corresponding to $\sin x = -\frac{1}{4}$ are $\pi - \theta_2$ and $2\pi + \theta_2$.
The sum of the four solutions is thus 4π, and this rounds to 13.