Nassau County Interscholastic Mathematics League
Team Contest Answers must integers between 0 and 999 inclusive. 2010 – 2011
Calculators are allowed.

1. If x % of x is 2.25, and $x > 0$, find x.

2. Find the sum of the three smallest composite numbers that have no prime factor less than 10.

3. The product of n distinct integers is 18. Find the maximum value of n.

4. Points A and D are on opposite sides of $\overleftrightarrow{BC}$, $\overline{AB} \perp \overleftrightarrow{BC}$, and $\overline{CD} \perp \overleftrightarrow{BC}$, AB = 9, BC = 21, and CD = 11. Find AD.

5. One of the roots of $x^2 + Bx + C = 0$, where B and C are integers, is $\sqrt{2} + 1$. Find $|B + C|$.

6. In triangle ABC, $AB = 5$, $BC = 6$, and $AC = 7$. Point O is outside the triangle and in the interior of angle A. A circle with center O is tangent to side $\overline{BC}$ at Q, and to sides $\overline{AB}$ and $\overline{AC}$ extended at P and R, respectively. Find BQ.

7. The polynomial $P(x)$ has degree 3 and leading coefficient 1. If $P(1) = 1$, $P(2) = 2$, and $P(3) = 3$, find $P(4)$.

8. In Δ□□□, AB = 13, BC = 14, and AC = 15. The angle bisector of ∡□ meets □□ in point D. A perpendicular line from vertex A meets □□ in point H. Find the product of the lengths of □□ and □□.

9. Let m be the second smallest positive integer x for which $\tan\left(\log_{10} 50x\right)^0 = \tan\left(\log_{10} 50\right)^0$. Find the number of digits of m.

10. In triangle ABC, $AB = 5$, $BC = 7$, and $CA = 6$. Point P is inside the triangle, and the distances from P to $\overline{AB}$ and $\overline{AC}$, are respectively 2 and 3. The distance from P to $\overline{BC}$ can be expressed as $\frac{a\sqrt{b} - c}{d}$, where a, b, c and d are positive integers, b is not divisible by the square of any prime, and a and d are relatively prime. Find $a + b + c + d$.

1. The given condition is equivalent to $(x/100)x = 2.25$. Multiply both sides by 100 to obtain $x^2 = 225$. Thus x = 15.

2. The smallest primes after 10 are 11, 13 and 17. The candidates for the three requested composites are $11^2 = 121$, $11 \cdot 13 = 143$, $11 \cdot 17 = 187$, and $13^2 = 169$. The requested sum is 121 + 143 + 169 = 433.

3. Notice that $18 = -1 \cdot 1 \cdot 2 \cdot 3 \cdot -3$, so $n = 5$.

4. Extend $\overline{AB}$ through B. Let E be the projection of D onto $\overline{AB}$ extended. In right triangle AED, $AD^2 = AE^2 + DE^2 = 20^2 + 21^2 = 841$, so $AD = 29$.

5. Let $x = \sqrt{2} + 1$. Then $x - 1 = \sqrt{2}$, so $(x-1)^2 = 2$. Thus $x^2 - 2x + 1 = 2$, and so $x^2 - 2x - 1 = 0$. Therefore $(B, C) = (-2, -1)$, so $|B + C| = 3$.

6. Let $BP = BQ = x$, and let $CQ = CR = y$. Then $x + 5 = AP = AR = y + 7$, and $6 = BC = BQ + QC = x + y$. Solve to obtain $BQ = x = 4$.

7. Notice that 1, 2, and 3 are roots of $P(x) - x = 0$. Thus $P(x) - x = (x - 1)(x - 2)(x - 3)$, so $P(4) - 4 = 3 \cdot 2 \cdot 1 = 6$, and $P(4) = 10$.

8. It is a nice fact that the altitude to the 14 side in a 13, 14, 15 triangle has length 12. (Use either the Pythagorean theorem or Heron's formula to verify.) So, AH = 12, BH = 5, and CH = 9. (Notice the Pythagorean triples.) Using the angle bisector theorem (An angle bisector of a triangle divides the side that it intersects into the ratio of the lengths of the other two sides of the triangle.): BD = 13k and DC = 15k for some positive integer k. 13k + 15k = 14. So, k = 0.5 and BD = 6.5. DH = 6.5 – 5 = 1.5. The required product is 18.

9. The given equation is equivalent to $\tan(\log 50 + \log x)^0 = \tan(\log 50)^0$ and is satisfied if and only if log⍰=*180*⍰ , where k is an integer. If k = 0, x = 1 and if k = 1, x = ***10180.*** Thus log m = 180, so $m = 10^{180}$, which has 181 digits.

10. The sum of the areas of triangles APB, BPC, and CPA equals the area of triangle ABC. Use Heron's Formula to find that the area of triangle ABC is $6\sqrt{6}$. Let x be the distance from P to $\overline{BC}$. Then $(1/2) \cdot 5 \cdot 2 + (1/2) \cdot 6 \cdot 3 + (1/2) \cdot 7x = 6\sqrt{6}$. Solve to obtain $x = \dfrac{12\sqrt{6} - 28}{7}$. Then $a + b + c + d = 53$.