Time: 10 minutes

1. Let m be the minimum value of a^{b^c} where $a, b,$ and c are distinct elements of $\{2,3,4\}$ Compute the remainder when m is divided by 1000.

2. Each vertex of a square whose area is 100 is on a side of a square whose area is 144. Let d be the distance from a vertex of the bigger square to the **nearest** vertex of the smaller square. If $d = p - \sqrt{q}$, where p and q are positive integers, compute $p + q$.

Time: 10 minutes

3. Let L be the least common multiple of 2, 3, 4, 5, 6, 7, 8, 9, and 10. Compute the remainder when L is divided by 1000.

4. The sum of the lengths of the diagonals of a rhombus is 28 inches and its area is 52 square inches. Compute the length of a side of the rhombus.

Time: 10 minutes

5. In $\Delta ABC, m\measuredangle A = 52°$ and $m\measuredangle B=62°$. Let $R, S,$ and T be points of tangency of the circle inscribed in ΔABC for sides $\overline{AB}, \overline{BC},$ and $\overline{CA}$ respectively. Compute the degree measure of $\measuredangle TRS$.

6. If x and y are positive real numbers such that $4x + 5y = 20$, compute the maximum value of xy.

Solutions for Contest #2

1. Note that $a^{b^c} = a^{(b^c)}$. Of the six possible expressions that can be made: $2^{3^4} = 2^{81}, 2^{4^3} = 2^{64}, 3^{2^4} = 3^{16}, 3^{4^2} = 3^{16}, 4^{2^3} = 4^8$, and $4^{3^2} = 4^9$, the least is $4^8 = 65{,}536$. So, the requested remainder is 536.

2. The four right triangles determined by the two squares are congruent. So, the sum of the lengths of the legs of one of the right triangles must equal the length of the larger square, 12. So, the lengths of the legs of each right triangle are d and $12 - d$. So, $d^2 + (12 - d)^2 = 10^2$. So, $d = 6 \pm \sqrt{14}$ and since $d < 12 - d,\ d = 6 - \sqrt{14}$. So $p = 6$ and $q = 14$. So, $p + q = 20$.

3. The prime factorization of the numbers are $2, 3, 2^2, 5, 2 \cdot 3, 7, 2^3, 3^2$, and $2 \cdot 5$. Thus, $L = 2^3 \cdot 3^2 \cdot 5 \cdot 7 = 2520$. So the requested remainder is 520.

4. For the rhombus, call the length of a side s and the lengths of the diagonals $d_1 = 2x$ and $d_2 = 28 - 2x$. The area of a rhombus is $\frac{1}{2}d_1d_2 = 52$. So, $\frac{1}{2}(2x)(28 - 2x) = 52,$ or $x(14 - x) = 26$ or $x^2 - 14x + 26 = 0$. So, $x = 7 \pm \sqrt{23}$, and $14 - x = 7 \mp \sqrt{23}$. Then, since the diagonals of the rhombus are perpendicular, use the Pythagorean Theorem to get $s = 12$.

5. Since the lengths of tangent segments to a circle from a common exterior point are congruent, $BR = BS$ and $RA = AT$. We use the facts that ΔBRS and ΔRAT are isosceles and the fact that the sum of the measures of the interior angles of a triangle is 180 to conclude that $m\measuredangle BRS = 59$ and $m\measuredangle ART = 64$. So, $m\measuredangle TRS = 180 - (64 + 59) = 57$.

6. Since $y = -\frac{4}{5}x + 4$, let $P(x) = x\left(-\frac{4}{5}x + 4\right) = -\frac{4}{5}x^2 + 4x$. The graph of $P(x)$ is a parabola with a maximum point at $x = -\frac{4}{(2)\left(-\frac{4}{5}\right)} = \frac{5}{2}$ and $P\left(\frac{5}{2}\right) = 5$.

Answers: 1. 536 2. 20 3. 520 4. 12 5. 57 6. 5