Time: 10 minutes

1. Compute $19.27^2 + 11.73^2 + (38.54)(11.73)$.

2. Point *A* is outside a circle and $\overline{AB}$ and $\overline{AC}$ are tangent to the circle at *B* and *C*, respectively. Points *P* and *R* are on $\overline{AB}$ and $\overline{BC}$, respectively, $\overline{PR}$ is tangent to the circle at *Q*, and *AB* = 20. Find the perimeter of triangle *APR*.

Time: 10 minutes

3. Find the sum of the coefficients in the expansion of $(3x-1)^4$

4. Point *A* is on a circle whose center is *O*, $\overline{AB}$ is tangent to the circle, *AB* = 6, *D* is inside the circle, *OD* = 2, $\overline{DB}$ intersects the circle at *C*, and *BC* = *DC* = 3. If *r* is the length of the radius of the circle, find r^2.

Time: 10 minutes

5. Chauncey goes up a flight of 8 stairs. With each step, he goes up either one or two stairs. Find the number of different sequences of steps he can take to the top. Note that leaping one stair and then two stairs is different from leaping two stairs and then one stair.

6. In convex quadrilateral *ABCD*, diagonal $\overline{AC}$ is perpendicular to diagonal $\overline{BD}$, *AB* = 10, *BC* = 5, and *CD* = 11. Find *AD*.

<u>Solutions for Contest #1</u>

1. The given expression is equal to $\left(19.27+11.73\right)^2 = 31^2 = 961$.

2. The perimeter of triangle *APR* is equal to AP + PR + RA = AP + PQ + QR + RA. Because tangent segments from the same point are congruent, the last sum is equal to AP + PB + CR + RA = AB + AC = 20 + 20 = 40.

3. The polynomial obtained when $\left(3x-1\right)^4$ is expanded is equal to $\left(3x-1\right)^4$ for all values of x. In particular, they are equal when x = 1. But substituting 1 for x in the polynomial yields the sum of the coefficients. Thus the sum of the coefficients is equal to $\left(3\cdot 1-1\right)^4 = 2^4 = 16$.

4. Extend $\overline{BD}$ through D until it intersects the circle again at E. Use Power of a Point (http://en.wikipedia.org/wiki/Power_of_a_point) to conclude that $BA^2 = BC\cdot BE$. Then $6^2 = 3\cdot BE$, so BE = 12, and then DE = 6. Let P and Q be the points where the diameter containing points O and D intersects the circle. Use Power of a Point again, this time to conclude that $DP\cdot DQ = DE\cdot DC$. Then $\left(r-2\right)\left(r+2\right) = 6\cdot 3$, so $r^2 = 22$.

5. Let *a* be the number of one-stair steps Chauncey takes, and let b be the number of two-stair steps. Then *a* + 2*b* = 8, so (*a*, *b*) = (8, 0), (6, 1), (4, 2), (2, 3), or (0, 4), and the number of sequences he can take are $\binom{8}{0}+\binom{7}{1}+\binom{6}{2}+\binom{5}{3}+\binom{4}{4}$ = 1 + 7 + 15 + 10 + 1 = 34. Alternatively, use recursion. Let u_n be the number of allowable sequences for a flight of n stairs. When Chauncey reaches the nth stair, he must have come from stair (n –1) or stair (n – 2). Thus $u_n = u_{n-1} + u_{n-2}$ for $n \ge 3$. Because $u_1 = 1$ and $u_2 = 2$, the next few values in the sequence are 3, 5, 8, 13, 21, and 34, so $u_8 = 34$.

6. Let Point *E* be the intersection of the diagonals. Then $AB^2 + CD^2 = AE^2 + BE^2 + CE^2 + DE^2 = AE^2 + DE^2 + BE^2 + CE^2 = AD^2 + BC^2$. Thus $10^2 + 11^2 = 5^2 + AD^2$, and *AD* = 14.

   Alternatively, use the Pythagorean Theorem: *□□2+□□2=□□2* *□□2+□□2=112* *□□2+□□2=52* *□□2+□□2=102* Add these four equations to get *2□□2+□□2+□□2+□□2=□□2+246* Re-write to get *2□□2+50=□□2+246*. So, AD =14.