Nassau County Interscholastic Mathematics League

Contest #2 Answers must be in simplest exact form, unless otherwise noted. 2009-2010
Calculators are allowed.

Time: 10 minutes

7. Steve plans to walk on a treadmill for 40 minutes. If he walks at a constant rate of 3.6 mph for 15 minutes, compute the rate in mph that he will have to walk for the remainder of his workout in order to have covered a total distance of 3 miles at the end of 40 minutes.

8. A cone is inscribed in a sphere of radius 25cm. The ratio of the height of the cone to the diameter of its base is 3:2. Compute the volume of the cone.

Time: 10 minutes

9. A positive integer, N, divided by 8 has a quotient with a remainder of 5. The quotient divided by 7 produces a quotient with a remainder of 3. When this new quotient is divided by 5, the result is a quotient with a remainder
of 2. Compute the least value for N.

10. The area of a rhombus is 336 cm^2. Its perimeter is 100 cm.
Compute the length of each of its diagonals.

Time: 10 minutes

11. In an election between Jones and Smith, Jones leads by 54% to 46%, with 56% of the votes counted. Compute the per cent of the remaining votes that Smith must win, in order to win the election with 52% of the vote.

12. Write an equation of the locus of points 5 units away from the line $4x - 3y = 6$ and passing through quadrant II.

7) In 15 minutes, he will have walked a distance of .9 miles.

He needs to complete 2.1 miles in 25 minutes. $\frac{21}{10} \div \frac{5}{12} = 5.04$

8) Let $x =$ the radius of the cone. The right triangle formed by the radius of the sphere,the radius of the base of the cone, and the segment of the altitude of the cone between the center of the sphere and the base of the cone has legs of $x, 3x - 25$, and a hypotenuse of 25. $x^2 + 9x^2 - 150x + 625 = 625 \rightarrow 10x^2 = 150x.$

$x = 15.$ the height of the cone is 45. $V = \frac{1}{3}\pi(15)^2(45) = 3375\pi.$

9) Let $N = 8x + 5,\ x = 7y + 3,\ \textit{and}\ \ y = 5z + 2.$ The smallest possible value for z is 0. Working backwards, $y = 2,\ x = 17,\ N = 141.$

10) Represent the diagonals of the rhombus as $2x,$ and $2y$. The diagonals of a rhombus are perpendicular bisectors of each other, and each side of the rhombus is 25.

$x^2 + y^2 = 625$ and $\frac{(2x)(2y)}{2} = 336 \rightarrow 2xy = 336$

$(x + y)^2 = 625 + 336 = 961 \rightarrow x + y = 31$

$(x - y)^2 = 625 - 336 = 289 \rightarrow x - y = 17;\ 2x = 48, 2y = 14.$

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11) Let x be the total number of voters and let y be the fraction of remaining voters that Smith needs to win.

$.56x(.46) + (.44x)\,y = .52x$

$.2576 + .44y = .52 \rightarrow .44y = .2624 \rightarrow\ y = .59\overline{63} = 59\frac{7}{11}\%$

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12) The given line has a slope of $\frac{4}{3}$ and a y-intercept of -2. The desired line must have the same slope and be at a distance of 5 units to left of the given line. Moving from the y-intercept 3 units up and 4 to the left, the point $(-4,1)$ is 5 units from the given line. In point-slope form the equation of the new line is

$y - 1 = \frac{4}{3}(x + 4).$