Nassau County Interscholastic Mathematics League

Contest #1   Answers must be in simplest exact form, unless otherwise noted. 2009-2010

No calculators are allowed.

Time: 10 minutes

1. Express as a common fraction in lowest terms: $0.5\overline{3} - 0.3\overline{12}$.

2. If $a = 3 + \sqrt{3}$, compute $a^2 + \frac{36}{a^2}$.

Time: 10 minutes

3. Find all values of x such that: $\left|x^2 - 9\right| = 3x - 5$.

4. Express in simplest terms:

$$(x-2)^5 + 10(x-2)^4 + 40(x-2)^3 + 80(x-2)^2 + 80(x-2) + 32$$

Time: 10 minutes

5. A circle of radius 6cm intersects a circle of radius 5cm at points A and B. If the distance between the centers of the circles is 9cm, compute AB.

6. Compute $\sqrt{11+\sqrt{72}} + \sqrt{11-\sqrt{72}}$.

Solutions for Contest #1

1) $0.5\bar{3} = \dfrac{8}{15}$, $\quad 0.3\overline{12} = \dfrac{103}{330} \qquad \dfrac{8}{15} - \dfrac{103}{330} = \dfrac{73}{330}$

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2) $$a^2 + \frac{36}{a^2} = \left(a + \frac{6}{a}\right)^2 - 12 = \left(3 + \sqrt{3} + \frac{6}{3+\sqrt{3}}\right)^2 - 12 = 6^2 - 12 = 24$$

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3) If $|x| \geq 3, \qquad x^2 - 9 = 3x - 5 \rightarrow x^2 - 3x - 4 = 0; (x-4)(x+1) = 0 \rightarrow x = 4$

If $|x| < 3,\ 9 - x^2 = 3x - 5 \rightarrow x^2 + 3x - 14 = 0; x = \dfrac{-3 \pm \sqrt{9+56}}{2} \qquad x = \dfrac{-3+\sqrt{65}}{2}$

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4) Let $a = x - 2$. Using the binomial theorem, the expression becomes

$$a^5 + 10a^4 + 40a^3 + 80a^2 + 80a + 32 = (a+2)^5 = x^5$$

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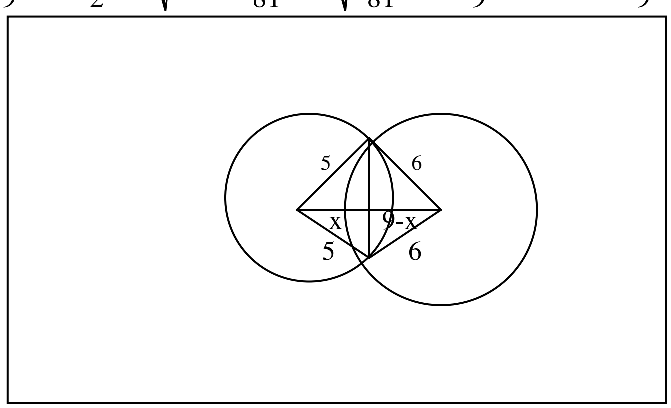
5) The line of centers of the circles is the perpendicular bisector of common chord $\overline{\text{AB}}$. Using the two right triangles,

$$25 - x^2 = 36 - (9-x)^2 \rightarrow 25 - x^2 = 36 - 81 + 18x - x^2$$

$$18x = 70,\ x = \frac{35}{9}. \quad \frac{\text{AB}}{2} = \sqrt{25 - \frac{1225}{81}} = \sqrt{\frac{800}{81}} = \frac{20\sqrt{2}}{9}; \text{AB} = \frac{40\sqrt{2}}{9}$$



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6) Let $a = \sqrt{11+\sqrt{72}}$ and $b = \sqrt{11-\sqrt{72}}$

$$(a+b)^2 = 11 + \sqrt{72} + 2\left(\sqrt{121-72}\right) + 11 - \sqrt{72} = 22 + 14 = 36 \qquad a + b = 6$$