

NASSAU COUNTY INTERSCHOLASTIC MATHEMATICS LEAGUE

Suggested Solutions

Contest #5

2008-2009

25. Let the numbers be  $x$  and  $x + 14$ . Then,  $\frac{x + x + 14}{2} = \sqrt{x(x + 14)} + 1$ ;  $x + 6 = \sqrt{x^2 + 14x}$ ;  
 $x^2 + 12x + 36 = x^2 + 14x$ ;  $x=18$ .

Answer: 32

26.  $\frac{\log 2^7}{\log 2^{-2}} = \frac{7 \log 2}{-2 \log 2} = -\frac{7}{2} = -3.5$ .

Answer: -3.5

27. The diagonal,  $d$ , of a rectangular prism with dimensions  $l$ ,  $w$ , and  $h$  is  $d = \sqrt{l^2 + w^2 + h^2}$ . Let the third dimension be  $h$ .  $h = \sqrt{26^2 - \left[ (4\sqrt{19})^2 + (8\sqrt{3})^2 \right]} = \sqrt{180}$

Answer:  $6\sqrt{5}$

28.  $399.91 = 400 - .09$  and  $159999.9919 = 160000 - .0081$ .

$$\frac{400 - 0.09}{160000 - .0081} = \frac{20^2 - (.3)^2}{20^4 - (.3)^4} = \frac{1}{20^2 - (.3)^2} = \frac{1}{400.09}$$

Answer:  $\frac{100}{40009}$

29. Drop an altitude to the side of length  $\sqrt{29}$ , separating the side into two segments of lengths  $x$  and  $\sqrt{29} - x$ . Let the altitude have length  $h$ . Then,  $h^2 = 17 - x^2$  and  $h^2 = 20 - (29 - 2x\sqrt{29} + x^2)$ .

$$17 - x^2 = -x^2 + 2x\sqrt{29} - 9 \text{ and } x = \frac{13}{\sqrt{29}}; h = \frac{18}{\sqrt{29}}. \text{ The area is } (0.5) \left( \frac{18}{\sqrt{29}} \right) (\sqrt{29}).$$

An altitude may be dropped to either of the other two sides. The answer yielded will be the same.

Answer: 9

30.  $\sin \theta \cos \theta = \frac{1}{4}$ ;  $\frac{1}{2} \sin 2\theta = \frac{1}{4}$ ;  $\sin 2\theta = \frac{1}{2}$ ;  $2\theta = 30^\circ$  or  $150^\circ$ .

Answer: 15 and 75 or  $15^\circ$  and  $75^\circ$