

NASSAU COUNTY INTERSCHOLASTIC MATHEMATICS LEAGUE

Suggested Solutions

Contest #3

2008-2009

13. The next greater numerical palindrome after 15951 is 16061, which means that the number of miles driven in the two-hour interval is 110, making the average speed 55 mph.

Answer: 55 or 55 mph

14. The graph of the equation is four line segments, consecutive pairs of which intersect at the points (10, 0), (0,10), (-10,0), and (0,-10). The figure formed is a square with a side of length  $10\sqrt{2}$  and whose area is 200. Also, the diagonal of the square has a length of 20. The area is equal to one-half the square of the diagonal.

Answer: 200

15.  $4a + 3b = 937 \Rightarrow b = \frac{937 - 4a}{3}$  is an integer when  $a = 1, 4, 7, \dots, 232$ . If you divide the last value

by the common difference between successive terms, you get 77 R 1. Thus, you have a total of 78 values. One can also use the arithmetic sequence formula:

$a_n = a_1 + (n - 1)d$ , where  $a_n = 232$ ,  $a_1 = 1$ , and  $d = 3$ . Solving for  $n$  yields 78

Answer: 78

16. Draw  $\overline{DE} \perp \overline{AB}$  and  $\overline{CF} \perp \overline{AB}$  so that  $AE = BF = 12$ . In rt  $\triangle DEB$ ,  $DB = 41$  and  $EB = 40$ , making  $DE = 9$ . In rt  $\triangle s$  ADE and BCF.  $AD = 15$  and  $BC = 15$ .

The area of the trapezoid is  $(0.5)(9)(28 + 52) = 360$  square units.

The perimeter of the trapezoid is  $28 + 15 + 52 + 15 = 110$  units.

Answer: 250

17.  $\frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy}$ , which is smallest when  $xy$  is at its greatest value. The rectangle with dimensions  $x$

and  $y$  has its greatest area when  $x = y$ . If  $x = y$ , then,  $\frac{1}{10.5} + \frac{1}{10.5} = \frac{2}{10.5} = \frac{4}{21}$ .

Answer:  $\frac{4}{21}$

18. The sum,  $S$ , of the first  $n$  consecutive positive integers is  $S = \frac{n(n+1)}{2}$ . If we adapt and extend this

formula to this problem,  $n = x + 8$  and  $n + 1$  (the sum of the first and last integer) =  $x - 7$ .

So,  $297 = \frac{(x+8)(x-7)}{2}$ ;  $x^2 + x - 56 = 594$ ;  $x^2 + x - 650 = 0$ ;  $(x - 25)(x + 26) = 0$ ;  $x = 25$  or  $-26$ .

Reject  $-26$  since it is not a part of the set of integers in the problem.

Answer: 25

Alt. Approach: The sum of the integers from  $-7$  through  $7$  is  $0$ . Thus,  $8 + 9 + 10 + \dots + x = 297$ .

Since  $1 + 2 + 3 + \dots + 7 = 28$ ,  $1 + 2 + 3 + \dots + x = 297 + 28$  and the sum,  $S$ , equals  $\frac{x}{2}(x+1) = 350$ .