

NASSAU COUNTY INTERSCHOLASTIC MATHEMATICS LEAGUE

Suggested Solutions

Contest #2

2008-2009

7. The second leg of the right triangle measures $\sqrt{(8\sqrt{5})^2 - (4\sqrt{11})^2} = \sqrt{320 - 176} = \sqrt{144} = 12$.

If the diagonal of the square is 12, then its side measures $6\sqrt{2}$ and its area is 72.

Answer: 72

8. 11^1 ends in 11; 11^2 ends in 21; 11^3 ends in 31; 11^4 ends in 41; 11^5 ends in 51; 11^6 ends in 61; 11^7 ends in 71; 11^8 ends in 81; 11^9 ends in 91; 11^{10} ends in 01. The pattern continues and is cyclic over every 10 consecutive positive integral powers of 11. Therefore, 11^{57} ends in 71.

Every integral power of 5 greater than or equal to 2 ends in 25.

Answer: 46

9. By the factor or remainder theorem, $f(-2) = 16 - 8a + 28 - 2b + 12 = 0$; $8a + 2b = 56$; $4a + b = 28$. Also, $f(3) = 81 + 27a + 63 + 3b + 12 = 0$; $27a + 3b = -156$; $9a + b = -52$. Subtracting $(4a + b = 28)$ from $(9a + b = -52)$ yields $5a = -80$, $a = -16$ and $b = 92$.

Answer: 108

10. Let $x = \#$ of hours they worked together.

Then, $\frac{x}{9} + \left(\frac{x}{9} + \frac{2x}{15}\right) + \frac{2}{15} \cdot \frac{3}{2} = 1$; Multiplying by 45 yields $5x + 5x + 6x + 9 = 45$; $x = 2.25$

Answer: 2.25

11. Let $S =$ the indicated sum. Then, $nS = A + \frac{A}{n} + \frac{A}{n^2} + \dots$. $nS - S = S(n - 1) = A$.

Answer: $\frac{A}{n-1}$

12. The ratio 5:9 is equivalent to $\frac{8}{14.4}$. Therefore, triangles ABD and BDC are similar. Let $CD = x$.

Then, $\frac{5}{8} = \frac{8}{x}$ and $x = 12.8$

Answer: 12.8