NASSAU COUNTY INTERSCHOLASTIC MATHEMATICS LEAGUE

Suggested Solutions **Contest #1** **2008-2009**

1. Let h = hundreds' digit, t = tens' digit, and u = units' digit. Then 100h + 10t + u = 11(h + t + u) or 89h = t + 10 u. The maximum value of t + 10u is 99. So, h must be 1, making t = 9 and u = 8.

Answer: 198

2. 5x + 2y = 2.88y or 5x = 0.88y. So, $\frac{y}{x} = \frac{5}{0.88} = \frac{500}{88}$.

Answer: $\frac{125}{22}$

3. Let x = # of games in the entire season and p = the average # of pts scored in remainder of season. Then, $18\left(\frac{1}{4}x\right) + 28\left(\frac{1}{3}x\right) + p\left(\frac{5}{12}x\right) = 26x$ **; Multiplying by 12 and dividing by x yields the equation 54 + 112 + 5p = 312. So, p = 29.2.**

Answer: 29.2

4. Let the legs to which the medians are drawn measure 2x and 2y, making $\sqrt{4x^2+4y^2}$ **or** $2\sqrt{x^2+y^2}$ **the length of the hypotenuse. Then, the two equations based on the Pythagorean theorem that can be written are:**

$$x^2 + (2y)^2 = 64$$
$$(2x)^2 + y^2 = 76$$

Therefore, $5(x^2 + y^2) = 140$; $x^2 + y^2 = 28$; $\sqrt{x^2+y^2} = 2\sqrt{7}$

Answer: $4\sqrt{7}$

Though it is not necessary to do so for this problem, it should be observed that if x and y were solved for individually, one of the right triangles formed when the medians are drawn has side lengths which determine it to be a 30-60-90 triangle.

5. Let x = # hrs Amber is on the road before Bobbie catches up to her. Since Bobbie leaves 18 minutes after Amber, Bobbie's time on the road is (x – 0.3) hrs. So, 4x = 5(x – 0.3) and x = 1.5. The total distance traveled before Amber and Bobbie are first at the same point on the route is 6 kilometers which they reach at 1:05 pm. Cheryl needs one hour to meet them. Thus, she should leave at 12:05 pm.

Answer: 12:05 pm (Do not accept 12:05)

6. f(2) = $\frac{3f(1)+1}{3} = \frac{22}{3}$. **f(3) =** $\frac{23}{3}$; **f(4) =** $\frac{24}{3}$; **The pattern indicates that f(n) =** $\frac{20+n}{3}$, **n = 1,2,3, ...**

So, f(n + 1) = $\frac{3[f(n)]+1}{3} = \frac{3\left(\frac{20+n}{3}\right)+1}{3} = 676.\overline{3}$; **n = 2008**

Answer: 2008