SAMPLE SOLUTIONS, Contest #6

T1. Answer: 15.2

The old cost of a barrel of oil is $\frac{x}{240}$ dollars. The new cost of a barrel of oil is $\frac{0.84x}{175}$ dollars.

$$\frac{\frac{0.84x}{175}}{\frac{x}{240}} = 1.152.$$

T2. Answer: 13.44

Let the length of the legs of the right triangle be x and y and the length of the hypotenuse be z. Then, $x^2 + y^2 + z^2 = 5000 \Rightarrow 2z^2 = 5000 \Rightarrow z = 50$. The longer leg has a length of 48 and the shorter leg has a length of 14. If h is the altitude to the hypotenuse, the area of the triangle is

$$\frac{1}{2}(14)(48) = \frac{1}{2}(50)h = 13.44$$

T3. Answer: 10

$$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2 = 96. \quad \left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 = 100. \quad \sqrt{\left(x + \frac{1}{x}\right)^2} = \left|x + \frac{1}{x}\right| = 10.$$

T4. Answer: 6

$-13 \le x^2 - 7x - 31 \le 13$; For $x^2 - 7x - 31 \le 13$, $-4 \le x \le 11$. For $x^2 - 7x - 31 \ge -13$, the solution set is $x \le -2$ or $x \ge 9$. The solution set to the compound absolute value inequality is the intersection of the solution sets for the two separate inequalities, $-4 \le x \le -2$ or $9 \le x \le 11$. The integers in this set are {-4, -3, -2, 9, 10, 11}.

T5. Answer: 11

Case 1. Right angle at A or $\overline{CA} \perp \overline{AB}$. The product of their slopes is $-\frac{c}{2} \cdot 5 = -1$; $c = \frac{2}{5}$.

Case 2. Right angle at C or $\overline{CB} \perp \overline{CA}$. The product of their slopes is $\frac{c-5}{-3} \cdot \frac{c}{-2} = -1$; $c = 2$ or 3.

Case 3. Right angle at B or $\overline{CB} \perp \overline{BA}$. The product of their slopes is
$\frac{c-5}{-3} \cdot 5 = -1$; $c = \frac{28}{5}$. $\quad c = \frac{2}{5} + \frac{28}{5} + 3 + 2 = 11.$

T6. Answer: 70.5

It is given that point O is the common center of the cube and the sphere. Let point P be the center of the bottom face of the cube. Draw $\overline{OP}$, $\overline{AP}$, and $\overline{BP}$. If s is the length of an edge of the cube, then, in rt $\triangle AOP$, $OP = \frac{1}{2}s$, $AP = \frac{1}{2}s\sqrt{2}$, and hypotenuse $AO = \frac{s}{2}\sqrt{3}$. Let $\theta = \angle AOB$. Then, by the law of cosines, $AB^2 = OA^2 + OB^2 - 2(OA)(OB)\cos\theta$.

So, $s^2 = \frac{3}{4}s^2 + \frac{3}{4}s^2 - 2 \cdot \frac{3}{4}s^2 \cdot \cos\theta$; $\cos\theta = \frac{-\frac{1}{2}s^2}{-\frac{3}{2}s^2} = \frac{1}{3}$. **The degree-measure of θ is 70.5.**