

SAMPLE SOLUTIONS, Contest #4

19. Answer: 37

Let T and B be Ted's and Bill's present age, respectively.

Then, $T = 3(B - 6)$ and $T + 10 + B + 10 = 5B - 13$. By substitution, $T = 27$. and $T + 10 = 37$.

20. Answer: 4

$3^1 = 3, 3^2 = 9, 3^3 = 27, 3^4 = 81$, after which the cycle repeats itself with respect to the units' digit. Thus, 3^{25} will have a units' digit of 3.

$8^1 = 8, 8^2 = 64, 8^3 = 512, 8^4 = 4096$, etc. So, 8^{31} will have a units' digit of 2.

$7^1 = 7, 7^2 = 49, 7^3 = 343, 7^4 = 2401$, etc. So, 7^{12} will have a units' digit of 1.

The units' digit for $3^{25} + 8^{31} - 7^{12}$ is $3 + 2 - 1 = 4$. When the quantity is divided by 10, the remainder will also be 4.

21. Answer: 202.5

Let x be the distance from city B to city C.

Then, $\frac{x}{45} + \frac{300-x}{65} = 6$; $65x + 13500 - 45x = 17550$; $x = 202.5$

22. Answer: 5.5

Let $x\%$ be the desired annual interest rate.

$$29000 \left(1 + \frac{x}{365}\right)^{270} = 30000 \left(1 + \frac{0.028}{365}\right)^{90}; \left(1 + \frac{x}{365}\right)^{270} = \frac{30}{29} \left(1 + \frac{0.028}{365}\right)^{90};$$

$$270 \log \left(1 + \frac{x}{365}\right) = \log 30 - \log 29 + 90 \log \left(1 + \frac{0.028}{365}\right); \log \left(1 + \frac{x}{365}\right) = 0.00006563540192;$$

$$1 + \frac{x}{365} = 10^{0.00006563540192}; x = 5.51670194$$

23. Answer: $\frac{9}{4}\pi$

Let r be the radius of circle O. Draw radii \overline{OA} and \overline{OB} . $\triangle AOP \sim \triangle BRP$.

$\frac{OA}{RB} = \frac{PA}{PB} = \frac{2}{8} = \frac{1}{4}$; $\therefore RB = 4r$. $\overline{OA} \perp \overline{PB}$ and $\overline{RB} \perp \overline{PB}$. From O, draw a segment perpendicular to \overline{RB} , meeting \overline{RB} at S. In rt $\triangle AOP$, $OS = 6$, $RS = 3r$, and $OR = 5r$.

$$(5r)^2 = (3r)^2 + 36; r^2 = \frac{9}{4}; \text{ the area of circle O is } \frac{9}{4}\pi.$$

24. Answer: 500 [Accept 500 mph.]

Let x be the speed in mph of the plane in still air on each leg of the trip.

$$\text{Then, } \frac{6240}{x-20} = \frac{6240}{x+20} + 1; 6240(x+20) = 6240(x-20) + (x+20)(x-20); x^2 = 250000; x = 500$$