SAMPLE SOLUTIONS, Contest #3

13. Answer: 149

The quantity x must be divisible by 7 and 11, the quantity y by 4 and 11 and the quantity z by 4 and 7. The least value of x is 77, of y is 44, and of z is 28 and the least value of $x + y + z$ is 149.

14. Answer: -56

$r^2 - 6r = -1$. Multiplying the factors two at a time yields $(r^2 - 6r + 8)(r^2 - 6r - 7)$. Thus, the required product is $(-1 + 8)(-1 - 7)$ or -56.

15. Answer: 56

Let S be the sum of the four original numbers.
Then, $\frac{S}{4} = \frac{S-20}{3} - 12$; 3S = 4S – 80 – 144; S = 224 and $\frac{S}{4}$ = 56.

16. Answer: 42

Let x be the number of seconds runner B requires to complete one lap. Without any loss of generality, assume the two runners start at the same point on the circular track. After 24 seconds, runner A completes $\frac{24}{56}$ of one lap and runner B completes $\frac{24}{x}$ of one lap.
$\frac{24}{56} + \frac{24}{x} = 1$; $\frac{3}{7} + \frac{24}{x} = 1$; $x = 42$.

17. Answer: 48

The given radical can be rewritten as $\sqrt[3]{18 \cdot 2^{11} + 15 \cdot 2^{11} + 9 \cdot 2^{11} + 12 \cdot 2^{11}} = \sqrt[3]{54 \cdot 2^{11}} = \sqrt[3]{27 \cdot 2^{12}}$. This yields an answer of $3 \cdot 2^4$ or 48.

18. Answer: $\frac{1}{4}$

You can use the LCM, 30, of the denominators of $\frac{1}{3}$, $\frac{2}{5}$, and $\frac{9}{10}$, the fractions mentioned in the problem as the universe of trips taken by the commuter. Thus, she took 10 trips in the compact car, arriving after 6:30 pm for 6 of the trips and 20 trips in the midsize car, arriving after 6:30 pm for 2 of the trips.
So, P(midsize car usage, given an arrival after 6:30 pm) = $\frac{2}{2+6}$ = $\frac{1}{4}$