

SAMPLE SOLUTIONS, Contest #1

1. Answer: 15

If D and d are positive integers and $D \geq d$, the maximum possible remainder for $D \div d$ is $d - 1$.

By trial and error: $99 \div 18 = 5R9$; $98 \div 17 = 5R4$;

$97 \div 16 = 6R1$; $79 \div 16 = 4R15$; 15 is 1 less than the divisor 16.

2. Answer: 20

Let l and w represent the lengths of a pair of adjacent sides of the rectangle. Then,

$1.15l(xw) = 0.92lw$; $1.15x = 0.92$; $x = 0.8$. Thus, the adjacent side must be reduced by $(1 - 0.8)\%$ or 20%

3. Answer: 12:08 pm

The minute hand of a standard 12-hour clock rotates at 6° per minute and the hour hand rotates at 0.5° per minute. Let x represent the number of minutes transpiring past noon. Then, $6x - 0.5x = 44$ or $60x - 5x = 440$ and $x = 8$.

4. Answer: $12\sqrt{3}$

Minor \widehat{AB} measures 120° . Draw a radius to point A and a segment from the center of the circle perpendicular to chord \overline{AB} , creating a 30-60-90 triangle. The radius (hypotenuse of the right triangle created) is 12 cm and $\frac{1}{2} \cdot AB$ is $6\sqrt{3}$.

5. Answer: 16

Let $x = \#$ of blue marbles and $x + 11 = \#$ of red marbles.

Then, $\frac{x+11}{2x+11} \cdot \frac{x+10}{2x+10} = 12 \left(\frac{x}{2x+11} \right) \left(\frac{x-1}{2x+10} \right)$;

$x^2 + 21x + 110 = 12x^2 - 12x$; $11x^2 - 33x - 110 = 0$; $x^2 - 3x - 10 = 0$; $(x - 5)(x + 2) = 0$; $x = 5$.

6. Answer: (3, 13, 1) [Accept $a = 3$, $b = 13$, $c = 1$.]

Let $x = \text{number}$. Then $x^2 = 29 + x$. Putting this equation in standard quadratic form and

applying the quadratic formula, $x = \frac{1 \pm 3\sqrt{13}}{2}$. Reject the negative root.