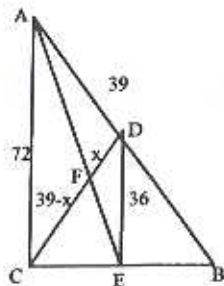


Nassau County Interscholastic Mathematics League
Solutions, Contest 4

#19. The square root of 600 is close to 25. Thus, the two pages across from each other in the middle of the pamphlet are numbered "24" and "25". From page 1 through page 24, there are 24 pages. The number of pages in the pamphlet is twice that, 48.

#20. In right $\triangle ACB$ below, C is the orthocenter. \overline{CD} is the median to the hypotenuse and its length is 39, one-half the length of the hypotenuse, 78. Medians \overline{AE} and \overline{CD} intersect at F , which is the centroid of $\triangle ACB$. When the midsegment \overline{DE} is drawn, DE is 36 since the segment joining the midpoints of two sides of a triangle is half as long as the third side, and $\triangle ACF$ and $\triangle EDF$, are similar. If we let $FD = x$ and $CF = 39 - x$, then $\frac{36}{72} = \frac{x}{39-x}$. So, $x = 13$ and $CF = 26$.



#21. The area of the circle is $\frac{16}{\pi^2} = \pi r^2$; So, $r = \sqrt{\frac{16}{\pi^3}} = \frac{4}{\pi\sqrt{\pi}}$; Thus, $d = \frac{8}{\pi\sqrt{\pi}} \approx 1.4367$.

#22. Factoring yields $\frac{(x^3+1)(x^3-1)}{x^3(x^2-1)+1(x^2-1)} = \frac{21}{5}$; $\frac{x^2+x+1}{x+1} = \frac{21}{5}$; $5x^2-16x-16=0$.
 $(5x+4)(x-4) = 0$; $x = -\frac{4}{5}$ or 4.

#23. The acceptable combinations are 6 valencias, 5 valencias and 1 temple, 5 valencias and 1 navel, 4 valencias and 2 temples, 4 valencias, 1 temple, and 1 navel, and 3 valencias and 3 temples.

So, the probability is

$$\frac{{}_8C_6 + 6 \cdot {}_8C_5 + 4 \cdot {}_8C_5 + {}_8C_4 \cdot {}_6C_2 + {}_8C_4 \cdot 6 \cdot 4 + {}_8C_3 \cdot {}_6C_3}{{}_{18}C_6} = \frac{317}{1326} \approx 0.2391$$

#24. Draw \overline{AE} and reflect the figure over \overline{BD} , producing an isosceles trapezoid. Draw the altitude of the trapezoid from A to F , F being a point on \overline{DE} between D and E . In right $\triangle AEF$, $AE = 37$, $EF = 12$, $AF = 35$. Let the reflection of E over \overline{BD} be E' . In right $\triangle AFE'$, $AE' = 91$, $AF = 35$, and $E'F = 84$. Let $AB = FD = x$. Then, $2x + 12 = 84$ and $x = 36$.