Nassau County Interscholastic Mathematics League

Solutions, Contest 3

#13.

Possible Digits	115	214	223	313	403	502	601	700	Total
N	3	6	3	3	4	4	4	1	28

N = # of acceptable permutations

#14. The number of diagonals of a polygon is given by

$${}_nC_2 - n = \frac{n(n-1)}{2} - n = \frac{n(n-3)}{2}.\ \text{So, } \frac{360}{n} = \frac{n(n-3)}{2} + 1;\ n^3 - 3n^2 + 2n = 720;\ (n-2)(n-1)n = 720.$$

Since n is an integer, the factors suggest three consecutive integers with a product of 720. By inspection, they are 8, 9, and 10. Alternately, one can construct a table and observe the pattern.

	# of diagonals	measure of an exterior angle
equilateral triangle	0	120
square	2	90
regular pentagon	5	72
regular hexagon	9	60
regular heptagon	14	360/7
regular octagon	20	45
regular nonagon	27	40
regular decagon	35	36 (35 + 1)

#15. A side of the square measures $5\sqrt{2}$ and a diagonal measures 10. The area of right $\triangle$ BFE is 100 and the area of right $\triangle$ BCD is 25. The area of trapezoid CDEF is the difference between these two areas, 75.

#16. Let x = number of pounds of water lost. Initially, the watermelon contained 36 lbs of water. $\frac{36-x}{45-x} = \frac{3}{5}$; x = 22.5; The new weight of the watermelon, in pounds, is (45 - 22.5) or 22.5, of which (36 – 22.5) or 13.5 lbs is water. 13.5 is 60% of 22.5.

#17. Mary reads 27 pages in 36 minutes. Beth reads 57 pages in 75 minutes.
27:36 = 3:4 and 57:75 = 19:25. If Mary reads 3 pages every 4 minutes, then she will take 456 minutes to read 342 pages. If Beth reads 19 pages every 25 minutes, then it will take her 450 minutes to complete the book, a difference of 6 minutes.

#18. For the front of the train to enter the tunnel and the rear of the train to exit the tunnel, the train must travel a total of (500 + 17,100) or 17,600 feet.

$$\frac{40 \text{ miles}}{1 \text{ hour}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}} \cdot \frac{5280 \text{ ft}}{1 \text{ mile}} = 3,520 \text{ ft per min. and } 17,600 \div 3,520 = 5.$$
