

**Nassau County Interscholastic Mathematics League
Solutions, Contest 1**

#1. $y = 2x - 3$ and $16x + 25 = 2(2x + 3)^2 + 7$.

$8x^2 - 40x = 0$ and $x = 0$ (reject) or 5 and $y = 7$. The perimeter is $2(7 + 105) = 224$.

#2. To average 81 on the first five quizzes, Adam must earn a total of 405 points. The highest possible total number of points on the fourth and fifth quizzes is 200. If x is the lowest possible score Adam can receive on the third quiz and still average 81 on the first five, then

$x = 405 - (84 + 76 + 200) = 45$.

#3. $\frac{x+y+70}{3} = \frac{y+z+58}{3} + 15$. So, $x + y + 70 = y + z + 58 + 45$ and $x - z = 33$.

#4. Given that point D is the foot of the altitude \overline{BD} , the number $\frac{a}{2} - 1$ is positive and vertex B of $\triangle OBC$ lies in Quadrant 1. The area of $\triangle OBC$ is $\frac{1}{2}a\left(\frac{a}{2} - 1\right) = 12$ and $a = 8$ or -6 (reject).

Therefore, the area of $\triangle OBD$ is $\frac{1}{2}\left(\frac{a}{2} + 1\right)\left(\frac{a}{2} - 1\right) = 7.5$.

#5. The GCF is 3^6 . $3^6(3^5 - 3^3 + 3 - 1) = 3^4 \cdot 3^2(243 - 27 + 3 - 1) = 81 \cdot 1962$.

#6. For the number to be divisible by 48, it must be divisible by 16 and by 3. The sum of the digits given is a number divisible by 3. Hence, the possibilities for the missing digit are 0, 3, 6, and 9. A number is divisible by 16 if the number formed by its last four digits is divisible by 16. By trial and error, the only digit that works in the hundreds' place is 3.
