Nassau County Interscholastic Mathematics League

Solutions **2004-2005**

Contest #4

T1. Only statements ii and iv are false, so 5 are true.

T2. Only the third and fifth have only positive real solutions. Thus, the probability is $\frac{2}{5}$ or 0.4 of selecting one equation meeting the conditions of the problem each time. Now, since we are picking with replacement, we have binomial probability, and the required probability is $\sum_{k=2}^{5} {}_5C_k \cdot \left(\frac{2}{5}\right)^k \left(\frac{3}{5}\right)^{5-k}$, which is about 0.663.

T3. $\log_2(x+2) - \log_2(x) = 3$, so $\log_2\left(\frac{x+2}{x}\right) = 3$, and $\frac{x+2}{x} = 8$, so $x = \frac{2}{7}$.

T4. By (1), Al does neither gymnastics nor basketball. By (3), Bob does not play football or do gymnastics, so the gymnast is Carl. Then Carl does not play football, so Al is the football player. By (2), since Carl is the gymnast, he doesn't play basketball, thus Bob plays basketball. By (4), Al can't be the soccer player. By (6), Carl doesn't play baseball, and Bob (the basketball player) doesn't play baseball, so Al must be the baseball player, and therefore he doesn't play tennis. By (7), Carl can't play soccer, so Bob plays soccer. Finally, Carl plays tennis. So Bob does soccer and basketball.

T5. Consider vectors $\overrightarrow{AB} = (3,-5,-1)$ and $\overrightarrow{AC} = (5,-1,3)$. Using the vector formula referenced in problem 12 earlier this year, $\cos\angle BAC = \frac{(3,-5,-1)\bullet(5,-1,3)}{|(3,-5,-1)||(5,-1,3)|} = \frac{17}{35}$. The angle is about 60.9°. Or find the lengths of all three sides of the triangle and use the Law of Cosines.

T6. The equation of the parabola is $y = \frac{x^2}{12} + 1$. The distance from (x, y) to the focus is the same as the distance from (x, y) to the directrix, which is $y + 2$. So $d(x) = y + 2 = \frac{x^2}{12} + 1 + 2 = \frac{x^2}{12} + 3$