**Nassau County Interscholastic Mathematics League**

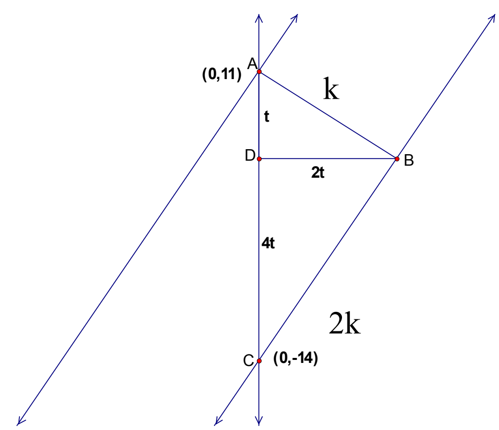
**Solutions** **2004-2005**

**Contest #3**

13. If Amanda is guilty, then Hugg (2) is true, so it's not Amanda. If Kiss is guilty, then Amanda (1) is true, so it's not Kiss. If Hugg is guilty, then only Hugg (2) is true, which works.

14. Each fraction is of the form $\frac{x^2-1}{x^2+x}=\frac{(x+1)(x-1)}{x(x+1)}=\frac{x-1}{x}$. So the product can be written

in the form $\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdots\frac{18}{19}\cdot\frac{19}{20}=\frac{1}{20}$




15. Method 1: A perpendicular to the lines through $(0,11)$ is $y=\frac{-1}{2}x+11$, which intersects the other line at (10,6), so the distance is the distance from (0,11) to (10,6) $=5\sqrt{5}$

Method 2: Refer to the figure shown. Draw $\overline{DB}$ horizontal. Let AD = t. Since $\overleftrightarrow{AB}$ has slope $-\frac{1}{2}$, BD = 2t. Now, since $\overleftrightarrow{CB}$ has slope 2, DC = 4t.Now, 5t = 25, so t =5, and right $\Delta ADB$ has legs 5, 10, so the hypotenuse is $5\sqrt{5}$.

Method 3: Refer to the figure shown. Draw $\overline{DB}$ horizontal. $\Delta ABC \sim \Delta ADB$ so the ratio of its sides is the slope ratio of the lines. $AB=k$, $CB=2k$ and AC=25 Using the Pythagorean theorem, $k=5\sqrt{5}$.

There is also a formula for the distance between two lines given their equations. Can you derive it?

16. $\overline{AB}$ has slope $\frac{4}{3}$, so the altitude from C has slope $-\frac{3}{4}$ and contains $(8,-2)$. So an equation is $y+2=\frac{-3}{4}(x-8)$. Changing this to slope intercept form, we get $y=\frac{-3}{4}x+4$.

17. Method 1: Factor (z + 3i)(z – 2i) = 0, so z = 2i or –3i
Method 2: use the Quadratic Formula

18. AB = BC =6, so CE = CD = AD = 6. So we have a trapezoid with bases 6, 12 and the altitude is the height of the equilateral triangle EDC, which is $3\sqrt{3}$, so the area is $\frac{1}{2}(6+12)\cdot 3\sqrt{3}=27\sqrt{3}$.

Alternate approach, divide into three equilateral triangles.

Another approach, compute the sum of the areas of parallelogram ABCD and equilateral triangle EDC.