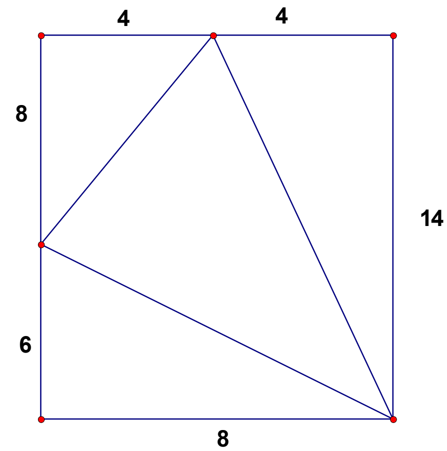
Nassau County Interscholastic Mathematics League

Solutions **2004-2005**

Contest #2

7. Method 1: Encase the triangle in a rectangle, using horizontal and vertical lines, as shown. The area of the triangle is the area of the rectangle minus the areas of the three right triangles, so the area is 112 – 16 – 28 – 24 = 44.



Method 2: The area is given by half the absolute value of the determinant of the matrix $\begin{vmatrix} -1 & 4 & 1 \\ 3 & 12 & 1 \\ 7 & -2 & 1 \end{vmatrix}$ = -88, so .5(88) = 44

Method 3: Draw an altitude, write its equation and the equation of the opposite side, find the point of intersection. Then find the lengths of the altitude and side, and use them to get the area. [Note: we don't recommend this method, but it works.]

8. Method 1: make a truth table.
Method 2: $(p \wedge q) \vee (p \wedge r)$ is equivalent to $p \wedge (q \vee r)$, which is the negation of the other expression, so they can't have the same truth value, hence they are the same 0 times.

9. The number of possible choices is ${}_{15}C_3 = \dfrac{(15)(14)(13)}{3!} = 455$. The sum of 10 can be gotten by choosing 1-2-7 or 1-3-6 or 1-4-5 or 2-3-5. $\dfrac{4}{455}$ is the probability.

10. By the altitude on hypotenuse theorem, $(MA)^2 = (MT)(MH)$. Let $x = MH$, so $16 = x(x + 6)$, and we get $x = 2$ or -8, but the length can't be negative, so MH = 2. Now note that all three triangles are 30-60-90 so the long leg is $4\sqrt{3}$. Or use the Pythagorean Theorem or altitude on hypotenuse to get $(AT)^2 = 48$, so $AT \approx 6.928$.

11. The factors can be $(x+1)(x+6), (x-1)(x-6), (x+2)(x+3)$, or $(x-2)(x-3)$, so the middle coefficient is $7, -7, 5$, or -5.

12. Method 1: Find the lengths of the sides and use Law of Cosines
Method 2: [using vectors] Vector AB = (4,8) and vector AC = (8,-6),
and $\cos\angle BAC = \dfrac{\text{dot product}}{\text{product of lengths}} = \dfrac{-16}{\left(4\sqrt{5}\right)(10)}$, so the angle is about 100.3°

Method 3: Find the area of the triangle by encasement, matrices or Hero's formula, find the lengths of sides $\overline{AB}$ and $\overline{AC}$, then area = (.5)(AB)(AC)(sin∠BAC) to get the angle